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The Conformable Derivative Is Used to Solve a Fractional Differential Equation Analytically

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ABSTRACT

In this paper, we talk about Fractional differential equations are generalizations of ordinary differential equations to an arbitrary (non-integer) order. Fractional differential equations have attracted considerable interest because of their ability to model complex phenomena. These equations capture nonlocal relations in space and time with power-law memory kernels. Due to the extensive applications of FDEs in engineering and science, research in this area has grown significantly all around the world. Almost the arrangement representation of fragmentary differential equation with distinctive conditions and deals with some methods for analytically solving the linear and non-linear of

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fractional differential equation based upon a conformable derivative by several methods and illustrate many example.

1. Introduction

In this paper, our focus is on the availability and uniqueness of solutions for linear and non-linear fractional differential equation.

$$D_\alpha(y)+h(t)y=k(t) \quad (1)$$

When $0<\alpha<1, y\in R^n$ and $D_\alpha(y)$ interpret the conformable derivative of y and $h,k:R\to R$ are α-differentiable Functions.

$$D_\alpha^{(2)}(y)+T(x)D_\alpha(y)+W(x)y=U(x) \quad (2)$$

When $T(x), W(x)$ and U(x) are α-differentiable functions and y is an unknown function.

In this paper, we discuss the availability of alternatives for the conformable fractional differential equation we give the solution of fractional differential equation both kinds homogenous and particular solutions according to our methods .

The paper is structured as follows. After introducing the basic definitions and theorems which are required to prove our main results, we presented the methods such as, order reduction method or Abel's formula, fractional equation with constant coefficients, Euler's equidimensional method (M. J. B. a. Z. A. Ilie, 2018), Variation of parameters, Undetermined coefficients (Horani, 2016), will be explained for solve fractional differential equation. In finally, we give some numerical example to illustrate our main results.

2. Preliminaries

Definition 2.1: Function given $h:[0,\infty)\to R$ then the conformable fractional derivative of h of order α is determined by

$$D_\alpha h(x)=\lim_{\epsilon\to 0}\frac{h(x+\epsilon x^{1-\alpha})-h(x)}{\epsilon}, \quad \forall x>0, \alpha\in(0,1) \quad (3)$$

Occasionally, write $h^\alpha(x)$ for $D_\alpha h(x)$ to indicate the conformable derivative of h of order α (Khalil R. e., 2014).

Theorem 2.1: Let $\alpha\in(0,1]$ and h,k be α-differentiable at a point $x>0$ then (Khalil R. e., 2014).

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1. $D_\alpha(ah+bk)=aD_\alpha(h)+bT_\alpha(k),\ \forall a,b\in R$
2. $D_\alpha(x^p)=px^{p-\alpha},\ \forall p\in R$
3. $D_\alpha(\lambda)=0,\ \forall\lambda\in R$
4. $D_\alpha(hk)=hD_\alpha(k)+kD_\alpha(h)$
5. $D_\alpha(\frac{h}{k})=\frac{kD_\alpha(h)-hD_\alpha(k)}{k^2}$
6. $D_\alpha(h)=x^{1-\alpha}\frac{dh}{dx}(x)$
7. $D_\alpha(h\circ k)(x)=x^{1-\alpha}h^\alpha(k(x))k^\alpha(x)$

Theorem 2.2: conformable fractional derivative of known functions

1. $D_\alpha(e^{bx})=bx^{1-\alpha}e^{ax}$
2. $D_\alpha(\sin(bx))=bx^{1-\alpha}\cos(bx),\quad b\in\mathbb{R}$
3. $D_\alpha(\cos(bx))=-bx^{1-\alpha}\sin(bx),\quad b\in\mathbb{R}$
4. $D_\alpha(\tan(bx))=bx^{1-\alpha}\sec^2(bx),\quad b\in\mathbb{R}$
5. $D_\alpha(\cot(bx))=-bx^{1-\alpha}\csc^2(bx),\quad b\in\mathbb{R}$
6. $D_\alpha(\sec(bx))=bx^{1-\alpha}\sec(bx)\tan(bx),\quad b\in\mathbb{R}$
7. $D_\alpha(\csc(bx))=-bx^{1-\alpha}\csc(bx)\cot(bx),\quad b\in\mathbb{R}$
8. $D_\alpha(\frac{1}{\alpha}x^\alpha)=1$
9. $D_\alpha(\sin(\frac{1}{\alpha}x^\alpha))=\cos(\frac{1}{\alpha}x^\alpha)$
10. $D_\alpha(\cos(\frac{1}{\alpha}x^\alpha))=-\sin(\frac{1}{\alpha}x^\alpha)$
11. $D_\alpha(e^{\frac{1}{\alpha}x^\alpha})=e^{\frac{1}{\alpha}x^\alpha}$

Definition 2.2: Let h be a continuous function. Then fractional integral of h is represent by:

$$J_\alpha^a h(t)=\int_a^t\frac{h(x)}{x^{1-\alpha}}dx \qquad (4)$$

Where $a\geq 0,\alpha\in(0,1)$ and the integral is the normal integral (Khalil R. e., 2014).

Example 2.1: find the following integral $j_{\frac{1}{2}}^{0}(\sqrt{t}cos(t))$

$$j_{\frac{1}{2}}^{0}(\sqrt{t}cos(t)) \rightarrow = \int_0^t \frac{\sqrt{x}cos(x)}{x^{1-\frac{1}{2}}} \rightarrow = \int_0^t cos(x)dx = sin(t)$$

Theorem 2.3: Let h be any continuous function in the domain of j_α (Sene, 2018).then

$$D_\alpha j_\alpha^a(h(t)) = h(t) \text{ , for } t \geq 0 \tag{5}$$

Proof: since, h is continues, then $j_\alpha^a f(t)$ is differentiable. So

$$D_\alpha(j_\alpha^a h(t)) = t^{1-\alpha}\frac{d}{dt}j_\alpha^a h(t)$$

$$= t^{1-\alpha}\frac{d}{dt}\int_a^t \frac{h(x)}{x^{1-\alpha}}dx = t^{1-\alpha}\frac{h(t)}{t^{1-\alpha}}$$

$$=\text{h(t)}$$

Theorem 2.4: Let $\alpha \in (0,1]$ and h is any continuous function in a domain of j_α , for $t > a$ we have

$$\frac{d}{dt}\left[j_\alpha^a f(t)\right] = \frac{h(t)}{t^{1-\alpha}} \tag{6}$$

This theorem is essential to obtain the analytical solution of conformable differential equations.

3. Main results

Definition 3.1: In general, differential equations of order α are considered mathematically represented by the following form: (M. J. B. a. Z. A. Ilie, 2018)

$$D_\alpha(y) + h(x)y = k(x) \tag{7}$$

When $0 < \alpha < 1, y \in R^n$ and $D_\alpha(y)$ denotes the conformable derivative of y and $h,k : R \rightarrow R$ are α-differentiable Functions, if $\alpha = 1$ we use the classical differential equations of first order shown like that $y' + h(t)(y) = k(t)$ We first take the case in which $k(t) = 0$, then

$$D_\alpha(y) + h(x)y = 0 \tag{8}$$

Is called the homogeneous If $k(t) \neq 0$ is called the non-homogeneous.

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Theorem 3.1: The homogeneous solution of the conformable differential equation (8) is indicated by

$$y_h(t) = ce^{-j_\alpha^0 h(t)} = ce^{r\frac{1}{\alpha}t^\alpha} \qquad (9)$$

When h is any continuous function in the domain of j_α^0. (Sene, 2018)

Proof: we have just verified that equation (8) is fulfilled by obtaining the function.

$y_h(x) = ce^{-j_\alpha^0 h(x)}$

By replacing into above equation and using theorem (2.4), we get

$$D_\alpha(y) + h(x)y = ct^{1-\alpha}\frac{d}{dx}\left[e^{-j_\alpha^0 h(x)}\right] + ch(x)e^{-j_\alpha^0 h(x)}$$

$$= -cx^{1-\alpha}\frac{d}{dx}\left[j_\alpha^0 h(x)\right]e^{-j_\alpha^0 h(x)} + ch(x)e^{-j_\alpha^0 h(x)}$$

$$= -cx^{1-\alpha}\frac{h(x)}{x^{1-\alpha}}e^{-j_\alpha^0 h(x)} + ch(x)e^{-j_\alpha^0 h(x)}$$

$$=0$$

Theorem 3.2: The particular solution of the conformable differential equation (7) is given by

$$y_p(t) = \lambda(t)e^{-j_\alpha^0 h(t)} \qquad (10)$$

where h is any continues function in the domain of j_α^0 and the function $\lambda : R \to R$ is obtained through the following condition

$$\lambda(t) = j_\alpha^0\left(k(x)e^{j_\alpha^0 h(x)}\right) \qquad (11)$$

Remark: The general candidate solution to the differential equations defined by (7) is given below $y(x)=y_h(x)+y_p(x)$

Example 3.1: find the particular and homogenous solution of the following differential equation (Khalil R. e., 2014)

$T_{\frac{1}{2}}(y) + \sqrt{x}y = xe^{-x}$

Solution: by theorem (3. 1) homogeneous solution is $y_h = ce^{-x}$

By theorem (3. 2) particular solution is $y_p = \lambda(x)e^{-j_\alpha^0 h(x)} = \frac{2}{3}x^{\frac{3}{2}}e^{-x}$

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Definition 3.2: Consider the general equation of the second order of fractional differential equation based on a conformable derivative as follows

$$D_{\alpha}^{(2)}(y)+T(x)D_{\alpha}(y)+W(x)y=U(x)\,. \qquad (12)$$

When $T(x)$, $W(x)$ and U(x) are α-differentiable functions and y is an unknown function. If $U(x)$ is zero, then fractional equation (12) reduces to the homogeneous equation

$$D_{\alpha}^{(2)}(y)+T(x)D_{\alpha}(y)+W(x)y=0 \qquad (13)$$

when $D_{\alpha}^{(2)}(y)=D_{\alpha}(D_{\alpha}(y))$.

Definition 3.3: The way for finding Wronskian of two functions $h(x)$ and $k(x)$ is given by (Horani, 2016)

$$\mathrm{W(h(x),k(x))}=\begin{vmatrix} \mathrm{h(x)} & \mathrm{k(x)} \\ D_{\alpha}h(x) & D_{\alpha}k(x) \end{vmatrix}=h(x)D_{\alpha}k(x)-k(x)D_{\alpha}h(x)$$

Definition 3.4: (Order reduction method) we assume that y_1 is a known nonzero solution of equation (13), $y_2=v\times y_1$ is a solution of equation (13), where v is unknown function

$$y_2=y_1 j_{\alpha}\left(\frac{1}{y_1^2}e^{-j_{\alpha}(T(x))}\right) \qquad (14)$$

This formula is called Abel's formula, the general solution of the homogeneous fractional differential equation of (13) is thus as follows

$$y_h=c_1y_1+c_2y_2 \qquad (15)$$

Example 3.2: Find another solution for the following fractional differential equation.

$2xD_{\frac{1}{2}}^{2}(y)+\sqrt{x}D_{\frac{1}{2}}(y)-2y=0$ Where $\mathrm{y_1=x}$ is solution

Solution: by the formula (14) we can get

$$2xD_{\frac{1}{2}}^{2}(y)+\sqrt{x}D_{\frac{1}{2}}(y)-2y=0$$

$$D_{\frac{1}{1}}^{2}(y)+\frac{1}{2\sqrt{x}}D_{\frac{1}{2}}(y)-\frac{1}{x}y=0$$

$$y_2=x\times j_{\frac{1}{2}}\left(\frac{1}{x^2}e^{-\frac{1}{2}ln(x)}\right)=-\frac{1}{2x}$$

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Definition 3.5: We are now discussion of the homogeneous Equation (12) on a special case where m and n are constant (M. J. B. a. Z. A. Ilie, 2018).

$$D_\alpha^2(y) + mD_\alpha(y) + ny = o \qquad (16)$$

For solve this fractional equation first we write as Characteristic equation of differential equation

$$r^2 + mr + n = 0 \qquad (17)$$

There are three kinds of situations:

Case 1: There are two real and distinct roots to the characteristic equation. r_1 *and* r_2, in this case $y_1 = e^{r_1 \frac{1}{\alpha}x^\alpha}$ and $y_2 = e^{r_2 \frac{1}{\alpha}x^\alpha}$ are Independent linear solution of (16), then the general solution is

$$y_h = c_1 e^{r_1 \frac{1}{\alpha}x^\alpha} + c_2 e^{r_2 \frac{1}{\alpha}x^\alpha} \qquad (18)$$

Case 2: The Characteristic equation has double solution $r = r_1 = r_2$ therefore the general solution is

$$y_h = (c_1 + c_2 \frac{1}{\alpha}x^\alpha)e^{r \frac{1}{\alpha}x^\alpha} \qquad (19)$$

Case 3: The characteristic equation has different root complex numbers, so they may be written in the form $a \pm ib$ and our two real solutions of the equation. (16) Are like below:

$$y_1 = (\cos b(\frac{1}{\alpha}x^\alpha))e^{a(\frac{1}{\alpha}x^\alpha)} \; and \; y_2 = (\sin b(\frac{1}{\alpha}x^\alpha))e^{a(\frac{1}{\alpha}x^\alpha)}$$

Then the general solution is

$$y_h = e^{a(\frac{1}{\alpha}x^\alpha)}\left(c_1 \cos b(\frac{1}{\alpha}x^\alpha) + c_2 \sin b(\frac{1}{\alpha}x^\alpha)\right) \qquad (20)$$

Example 3.3: Determine the solution to the following differential equation (Sene, 2018).

$$D_{\frac{1}{2}}^2(y) - 3D_{\frac{1}{2}}(y) + 2y = 0$$

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Solution: by case 1 solution is given by

The characterizing equation is like that $r^2-3r+2=0$, and have two distinct root, $r_1=1$ and $r_2 = 2$ the application of case 1, the solution is given by

$$y_h = Ae^{-\sqrt{t}} + Be^{-2\sqrt{t}}$$

Definition 3.6: (Euler's fractional equidimensional equation) the differential fractional homogeneous equation (M. J. B. a. Z. A. Ilie, 2018).

$$(\frac{1}{\alpha}x^{\alpha})^2 D_{\alpha}^2(y(x)) + m(\frac{1}{\alpha}x^{\alpha})D_{\alpha}(y(x)) + ny(x) = 0, x>0. \quad (21)$$

Where m,n constants are referred to as Euler's fractional equation, using the independent change variable. $z=\ln(\frac{1}{\alpha}x^{\alpha})$ We have

$$D_{\alpha}(y(z)) = (\frac{1}{\alpha}x^{\alpha})^{-1}\frac{dy}{dz} \quad (22)$$

$$D_{\alpha}^2(y(z)) = -(\frac{1}{\alpha}x^{\alpha})^{-2}\frac{dy}{dz} + (\frac{1}{\alpha}x^{\alpha})^{-2}\frac{d^2y}{dz^2} \quad (23)$$

Substituting Equation (22) and (23) into Equation (21), results in

$$\frac{d^2y}{dz^2} + (m-1)\frac{dy}{dz} + ny = 0 \quad (24)$$

This equation (24) is a standard constant coefficient differential equation, and based on this approach, the auxiliary equation has the following form.

$$r^2 + (m-1)r + n = 0 \quad (25)$$

Suppose for solve equation (25), we have three case:

Case 1: Suppose r_1 *and* r_2 are roots of Equation (25). If these are separate real numbers, then the next solution 0f (21) can be achieved.

$$y_h = c_1(\frac{1}{\alpha}x^{\alpha})^{r_1} + c_2(\frac{1}{\alpha}x^{\alpha})^{r_2} \quad (26)$$

Case 2: If $r_1 = r_2$, we derive

$$y_h = \left(c_1 + c_2\ln(\frac{1}{\alpha}x^{\alpha})\right)(\frac{1}{\alpha}x^{\alpha})^{r_1} \quad (27)$$

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Case 3: if r_1 *and* r_2 has different root complex numbers, then the general solution of equation (21) will be derived as follows

$$y_h = (\frac{1}{\alpha}x^\alpha)^a\left[c_1 cosb\ln(\frac{1}{\alpha}x^\alpha) + c_2 sinb\ln(\frac{1}{\alpha}x^\alpha)\right] \qquad (28)$$

Example 3.4: We consider the following homogenous equation.

$$(\frac{1}{\alpha}x^\alpha)^2 D_\alpha^2(y(x)) - 2(\frac{1}{\alpha}x^\alpha)D_\alpha(y(x)) + 2y(x) = 0$$

Solution: There is an auxiliary equation. $r^2 - 3r + 2 = 0$, Solutions are $r_1 = 1$ and $r_2 = 2$ therefore the answer is

$$y_h = c_1(\frac{1}{\alpha}x^\alpha) + c_2(\frac{1}{\alpha}x^\alpha)^2$$

Definition 3.7 :(Variation of parameters) Assume that y_1, y_2 are two solutions linearly independent homogenous fractional differential equation of the second order fractional differential equation (13), it is assumed that the particular is (Horani, 2016).

$$y_p = -y_1 j_\alpha\left(\frac{y_2 U(x)}{W_\alpha(y_1, y_2)}\right) + y_2 j_\alpha\left(\frac{y_1 U(x)}{W_\alpha(y_1, y_2)}\right) \qquad (29)$$

Example 3.5: Consider the fractional equation below

$2xD_{\frac{1}{2}}^2(y(x)) - \sqrt{x}D_{\frac{1}{2}}y(x) - 2y(x) = 4x^3$ Where the homogeneous solutions are $y_1 = x$ and $y_2 = -\frac{1}{2x}$

Solution: we simplify the equation and substitution in the formula

$$D_{\frac{1}{2}}^2(y(x)) - \frac{1}{2}x^{-\frac{1}{2}}(D_{\frac{1}{2}}y(x)) - \frac{1}{x}y(x) = 2x^2$$

$$y_p = -y_1 j_\alpha\left(\frac{y_2 U(x)}{W_\alpha(y_1, y_2)}\right) + y_2 j_\alpha\left(\frac{y_1 U(x)}{W_\alpha(y_1, y_2)}\right)$$

$$y_p = -x\int \frac{-x}{x^{-\frac{1}{2}}x^{\frac{1}{2}}}dx + (-\frac{1}{2}x^{-1})\int \frac{2x^3}{x^{-\frac{1}{2}}x^{\frac{1}{2}}}dx = \frac{x^3}{4}$$

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Definition 3.8: (Indeterminate coefficients) is a procedure for determining y_p when (12) is in form (M. J. B. a. Z. A. Ilie, 2018).

$$D_\alpha^{(2)}(y) + mD_\alpha(y) + ny = f(x) \qquad (30)$$

Where m, n are constant and f(x) is

$$f(x) = \left(a_0 + a_1(\frac{x^\alpha}{\alpha}) + a_2(\frac{x^\alpha}{\alpha})^2 + \cdots + a_n(\frac{x^\alpha}{\alpha})^n\right) e^{\beta(\frac{x^\alpha}{\alpha})} sin\gamma(\frac{x^\alpha}{\alpha})$$

Or

$$f(x) = \left(a_0 + a_1(\frac{x^\alpha}{\alpha}) + a_2(\frac{x^\alpha}{\alpha})^2 + \cdots + a_n(\frac{x^\alpha}{\alpha})^n\right) e^{\beta(\frac{x^\alpha}{\alpha})} cos\gamma(\frac{x^\alpha}{\alpha})$$

A particular solution is chosen from the form below

$$y_p = \Big[\left(A_0 + A_1(\frac{x^\alpha}{\alpha}) + A_2(\frac{x^\alpha}{\alpha})^2 + \cdots + A_n(\frac{x^\alpha}{\alpha})^n\right) e^{\beta(\frac{x^\alpha}{\alpha})} sin\gamma(\frac{x^\alpha}{\alpha}) + \left(B_0 + B_1(\frac{x^\alpha}{\alpha}) + B_2(\frac{x^\alpha}{\alpha})^2 + \cdots + B_n(\frac{x^\alpha}{\alpha})^n\right) e^{\beta(\frac{x^\alpha}{\alpha})} cos\gamma(\frac{x^\alpha}{\alpha})\Big](\frac{x^\alpha}{\alpha})^m \qquad (31)$$

Example 3.6: identify the general solution of the following equation using indeterminate coefficients.

$$D_{\frac{2}{3}}^2(y) - 2D_{\frac{2}{3}}(y) = 18\sqrt[3]{x^2} - 10$$

Solution: the characteristic equation is $r^2 - 2r = 0 \rightarrow r(r-2) = 0 \rightarrow \quad r = 0 \quad and \quad r = 2$

$$y_h = c_1 + c_2 e^{\frac{4}{3}\sqrt[3]{x^2}}$$

For finding y_p we use the formula

$$y_p = \frac{1}{\frac{2}{3}} x^{\frac{2}{3}}(A_1 x^{\frac{2}{3}} + A_0) = \frac{1}{\frac{2}{3}} A_1 x^{\frac{4}{3}} + \frac{1}{\frac{2}{3}} A_0 x^{\frac{2}{3}}$$

$$D_{\frac{2}{3}}(y_p) = 3A_1 x^{\frac{2}{3}} + A_0$$

$$D_{\frac{2}{3}}^2(y_p) = 2A_1$$

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We substitute $y_p,\ D_{\frac{2}{3}}(y_p),\ D^2_{\frac{2}{3}}(y_p)$ in the basic equation for finding constant

$$y_p = \left(2 - \frac{9}{2}x^{\frac{2}{3}}\right)\frac{x^{\frac{2}{3}}}{\frac{2}{3}}$$

So the general solution is

$$y_g = y_h + y_p$$

$$y_g = c_1 + c_2 e^{\frac{4}{3}\sqrt[3]{x^2}} - \left(2 - \frac{9}{2}x^{\frac{2}{3}}\right)\frac{x^{\frac{2}{3}}}{\frac{2}{3}}$$

4. Conclusion

In this article, explain briefly some methods for solving fractional deferential equations. It is difficult to determine the general form of the conformable differential equations. We know that there are numerous solutions to the conformable differential equation. This article helps to provide the explicit form of the conformable differential equation's candidate solution. These methods have been presented. We get an exact solution as a result, so there is no need to use a numerical method.

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داتاشراوی سازگار بەکاردێت بۆ شیکارکردنی هاوکێشەی داتاشراوی کەرتی بە شێوەی ئانالایزکردن

پـوختـە:

لەم توێژینەوەیەدا باس لە هاوکێشەکانی داتاشراوی کەرتی ئەکەین کە شێوەی گشتی داتاشراوی ژمارەی تەواوە، ئەم جۆرە داتاشراوە لەبەر ئەوەی توانایکی باشی هەیە لە مودێلەکانی تێکەل زانایان زۆر رویان تێ کردوە لەبەر ئەوەی ئەم جۆرە داتاشراوە توانای خەزن کردنێکی باشی هەیە هەروەها لەبەر بەکارهێنانەکانی لە بواری علوم وە هەندەسە لێکۆلینەوە لەم بوارە بە شیوەیکی بەرچاو لە سەرتاسەری جیهان گەشەی سەندوە. هەروەها باس لە هەندێ یاسا وە مێتود ئەکەین بۆ شیکارکردن وە ئەنالایزکردنی هاوکێشەی داتاشراوی کەرتی هێلی وە نا هێلی بە پێی داتاشراوی سازگار وە بەکارهێنانی چەند مێتودێک وە پێشان دانی ژمارەیک نمونە.

يتم استخدام المشتق المطابق لحل معادلة تفاضلية كسرية تحليليًا

الملخص:

في هذا البحث نتحدث عن المعادلات التفاضلية الكسرية وهي تعميمات للمعادلات التفاضلية العادية لترتيب عشوائي (غير صحيح) .جذبت المعادلات التفاضلية الكسرية اهتمامًا كبيرًا بسبب قدرتها على نمذجة الظواهر المعقدة .تلتقط هذه المعادلات العلاقات غير المحلية في المكان والزمان باستخدام نوى ذاكرة قانون السلطة .نظرًا للتطبيقات الواسعة لـ FDEs في الهندسة والعلوم، فقد نما البحث في هذا المجال بشكل كبير في جميع أنحاء العالم .تقريبًا تمثيل ترتيب المعادلة التفاضلية المجزأة بشروط مميزة ويتعامل مع بعض الطرق للتحليل التحليلي للمعادلة التفاضلية الخطية وغير الخطية بناءً على مشتق مطابق بعدة طرق وتوضيح العديد من الأمثلة.