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Comparison of Efficiency and Sensitively between (Three Sigma, Six Sigma and Median Absolute Deviation) by using X-bar Control Chart

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ABSTRACT

The main objective of any productivity operation is to get high quality materials and conform to the specifications so as to meet consumer desires. The quality is a principle in which some may think it is new, but it is as old as human, because the man was still in a permanent search for a good thing, and paying attention to the quality .Quality has great care in the industrialized developed countries being of economies exporter seek to control the foreign markets, scale estimators are very important in many statistical applications. The most common of scale estimators is the sample standard deviation. Standard deviation is non-robust to slight deviations from normality. The present study introduces the Comparison between another robust measure of dispersion namely Modified (**Three Sigma, Six Sigma and Median Absolute Deviation**) based on Shewhart mean control chart. When we compare robust methods, six sigma method have better property than the other robust methods, because control chart based on six sigma have the smallest control interval and also have the most number of out of control points.

I. INTRODUCTION

A control chart is an important aid or statistical device used for the study and control chart of the repetitive Process. Control charts were developed by Dr.W.A. Shewhart and it is based upon the fact that variability does exist in all the repetitive processes. Specifying the control limits is the most important step in designing a control chart. When the limits are narrow, the risk of a point falling beyond the limits increases, and hence increase the false indication that the process is out of control. If the limits are wider, the risk increases the points falling within the limits, falsely indicates that the process is in control[1] . There are many measures of scale available in literature and many control charts are available to control process dispersion or the dispersion of quality characteristics.

Quality Control Charts[2] [3]

A quality control chart (also called process chart) is a graph that shows average for the data (output) or the product fall within the common or normal range of variation if the process is under statistical control. Quality control charts were first invented by Walter A. Shewhart, and developed by him and his associate. He published a complete exposition of control charts in 1931. Which used by Shewhart in the construction of his charts. He concluded that a distribution can be transformed into a normal shape by estimating its mean and standard deviation. Shewhart`s idea was whether the production process is going well and naturally and the points plotted on the chart follow a normal distribution. For these reasons, Shewhart resorted to use the normal distribution in the construction of his charts.

Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is M_w and the Standard deviation of w is S_w . Then the center line, the upper control limit, and the lower control limit become. [3]

$$CL = T = M_w$$
$$UCL = M_w + 3\hat{\sigma}_w$$
$$LCL = M_w - 3\hat{\sigma}_w$$

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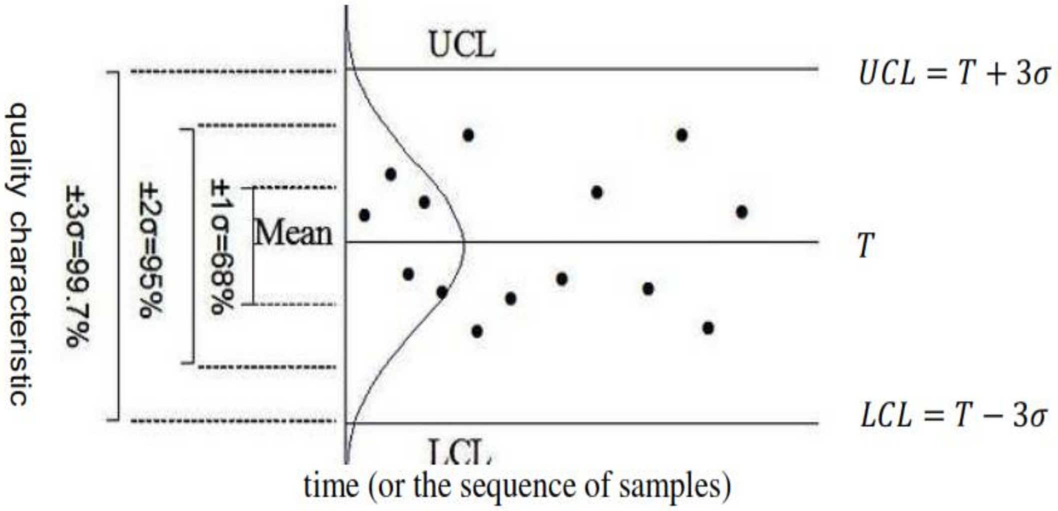

Figure 1: Normal distribution of continuous variables[3]

Classification of control charts[2] [3]

Control charts may be classified into two main types, which are:

1. **VARIABLE CONTROL CHARTS**

These charts are used in process control of products when the items produced are measurable (in one of the units of measurement).

The most important types of variable control charts can be divided into two types:

1.1 Shewhart Variable Charts: the most familiar Shewhart charts are

a. Average – Chart (or $\bar{x}$- chart)
b. Standard Deviation- Chart (S-chart)
c. Range - Chart (R-chart)
d. Individual – Chart(X- chart).
e. Median chart (Me- chart).

2.1 Non-Shewhart Variable Charts: the most popular

❖ **Classic Variable Charts:**[2] [3]

a. Cumulative sum control chart(CUSUM-chart)
b. Moving average chart (MA-chart)
c. Moving range chart (MR-chart)
d. Geometric Moving average chart (GMA-chart)

2. Attributes Control Charts

Attribute control charts are used when:
(a) Measurements are not possible (e.g., defect such as dented cans).
(b) Measurements are not practical (e.g., lengthy chemical analyses of raw products).

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(c) Several characteristics are combined on one chart (e.g., counts of different kinds of defects). In this case, the various characteristics can be lumped together into a single chart, or at most two or three charts, each covering that group of characteristics which reflects their importance such as minor, major, and critical.[3]

The attribute control charts can be classified into: [2]

a. Defective or nonconforming chart. p-chart (fraction nonconforming)
b. np-chart (number nonconforming).
c. Defects or nonconformities charts. C-chart (number of nonconformities).
d. U-chart (average number of nonconformities).

The following are charts used in this paper:

- **X-Bar chart based on Three sigma**(3σ)

Use x-Bar charts to monitor the changes in the mean of a process,

The center line of the x-Bar chart is calculated as follows:

$$\bar{\bar{x}} = \sum_{1}^{m} \bar{x}_j / m$$

The control limits of the mean chart based on three sigma is calculated as follows:

$$UCL = \bar{\bar{x}} + 3\hat{\sigma}$$
$$LCL = \bar{\bar{x}} - 3\hat{\sigma}$$

- **X-Bar chart based on Median Absolute Deviation**

The Median Absolute Deviation from the sample median (MAD) is a very robust scale estimator than the sample standard deviation [1]. It measures the deviation of the data from the sample median. The MAD is often used as an initial value for the computation of more efficient robust estimators. Let $x_{1,} x_2 \ldots x_n$ be a simple random sample of size n observation taken over (m).

MAD = 1.4826MD {|Xi - MD|} .....(2)

Where, MD is the median of $x_{1,} x_2 \ldots x_n$. The average of the MAD is computed using:

$$\overline{MAD} = \sum_{i=1}^{m} MAD / m \; .... (3)$$

When MAD is used as an estimate of variability, then MAD will be used as a replacement of the standard deviation (S), thus, $\hat{\sigma} = b_n \; \overline{MAD}$ [1].The control limits and central line for the mean control chart based on the MAD are calculated as follows:

UCL= $\bar{\bar{X}} + 3b_n \; \overline{MAD}$

CL= $\bar{\bar{X}}$ .... (4)

LCL= $\bar{\bar{X}} - 3b_n \; \overline{MAD}$

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Let A5 = $3b_n$, then the control limits in Eq. 4 will be reduced to Eq.:

UCL= $\bar{\bar{X}} + A_5\ \overline{MAD}$

CL= $\bar{\bar{X}}$ .... (5)

LCL= $\bar{\bar{X}} - A_5\ \overline{MAD}$

The values of (A_5) are computed and are presented in Table (A) in the appendix for various values of n.

❖ **X-bar chart based on Six Sigma** (6σ)

Six Sigma has been successfully applied in other manufacturing sectors. Developed by Motorola in 1983, from the statistical point of view, the term six sigma is defined as having less than 3.4 defects per million opportunities (DPMO) or a success rate of 99.9997% where sigma is a term used to represent the variation about the process average. If an organization is operating at three sigma levels for quality control [5], this is interpreted as achieving a success rate of 93% or 66,800 defects per million opportunities. Therefore, the six sigma method is a very rigorous quality control concept where many organizations still performs at three sigma level. The generating unit. Will include defining the importance of the quality of critical elements as defined by the consumer, the reduction to the minimum deviation in the operations and improve the capacity and increase the consistency and stability of the manufacturing process, As well as systems that help to achieve the objectives of 6 Sigma design. If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart, then no point fall outside the control limits because of the improvement in the quality of the process. Which adopt Six Sigma Initiatives [5]:

Concepts and Terminologies [5]**:**

1/ Upper Specification Limit (USL)

2/ Lower Specification Limit (LSL)

3/ Tolerance Level (TL): It is the difference between USL and LSL, TL = USL-LSL

4/ Process Capability (C_P **)**: This is the ratio of tolerance level to six times standard deviation of the process.

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

5/ Subgroup Size (n):

In order to make control chart analysis effective, it is essential to pay due regard to the rational selection of the subgroups. It is the choice of the sample size n and the frequency of sampling. It is also the number of observed values in any given sample or subgroup.

6/ Quality Control Constant

To construct the Six Sigma based control limits the Quality Control constants such as are introduced in this thesis.

$$L_{6\sigma} = \frac{4.831}{\sqrt{n}}$$

7/ Calculate ($\sigma_{6\sigma}$ **) and construction six sigma chart**

In this section a procedure to construct a six sigma based control chart for mean. Fix the Tolerance level (TL) and process capability (C_P) to determine the process standard deviation

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($\sigma_{6\sigma}$). Apply the value of $\sigma_{6\sigma}$ in the control limits $\bar{\bar{X}} + L_{6\sigma}\sigma_{6\sigma}$, to get the Six Sigma based control limits for Mean using Range. The value of $L_{6\sigma}\sigma_{6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 3.4*10^{-6}$ and z is a standard normal variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $C_p = \frac{TL}{6\sigma}$ presented in Table (B) for various combinations of TL and C_P. Further the value of $L_{6\sigma}$ is also obtained using the procedure given above and are presented in Table (C), for different values of n. The six sigma based control limits for mean using range are constructed as:

$$UCL = \bar{\bar{X}} + L_{6\sigma}\sigma_{6\sigma}$$

$$CL = \bar{\bar{X}}$$

$$LCL = \bar{\bar{X}} - L_{6\sigma}\sigma_{6\sigma}$$

Application

The example provided by Mohammed, S.O. (2013, Page No. 45) is considered here. The following data are the results of width (mm) of (PP Box strapping, Plastic Maudling components, and corrugated Boxes) bases for samples of five measurements.

(For width = 12mm)
Data collection (width)
Table 1: width of (PP Box strapping, Plastic Maudling components, and corrugated Boxes).

Sample no.	Sample observation					Mean	DAM
1	11.55	11.91	11.43	11.3	11.42	11.522	0.177912
2	11.57	12.15	11.16	12.18	11.65	11.742	0.726474
3	11.66	11.8	12	11.76	11.55	11.754	0.14826
4	12.29	11.8	12.22	12	11.88	12.038	0.29652
5	12.2	11.87	12.2	12.55	12.25	12.214	0.07413
6	11.8	11.85	11.75	12.15	12.05	11.92	0.14826
7	11.6	11.55	11.6	11.55	11.6	11.58	0
8	11.55	11.6	11.55	11.6	11.55	11.57	0
9	11.6	11.55	11.6	11.55	11.6	11.58	0
10	11.55	11.6	11.55	11.6	11.55	11.57	0
11	11.6	11.55	11.6	11.55	11.6	11.58	0

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12	11.55	11.6	11.55	11.6	11.55	11.57	0
13	11.57	11.55	11.16	11.16	11.65	11.418	0.14826
14	11.8	11.66	11.76	11.55	12	11.754	0.14826
15	12.05	12	11.8	11.66	11.55	11.812	0.29652

❖ **Construction of Control Limits three Sigma for Shewhart (X-Bar Chart)**

CL= $\bar{\bar{X}} + 3\hat{\sigma} = 11.70 + 3(0.216) = 12.36$

CL= $\bar{\bar{X}} = 11.70$

LCL= $\bar{\bar{X}} - 3\hat{\sigma} = 11.70 - 3(0.216) = 11.06$

❖ **Construction of Control Limits (MAD) for X-Bar Chart**

UCL= $\bar{\bar{X}} + A_5 \overline{MAD} = 11.70 + (3.618)(0.144) = 12.230$

CL= $\bar{\bar{X}} = 11.70$

LCL= $\bar{\bar{X}} - A_5 \overline{MAD} = 11.70 - (3.618)(0.144) = 11.186$

(From the Table A, $A_5 = 3.618$ for sample size n =5).

❖ **Construction of Control Limits Six Sigma for X-Bar Chart**

For a given (TL =USL - LSL = 12.55 – 11.16= 1.39) & CP = 1.45, it is found from the Table– B that the value of $(\sigma_{6\sigma})$is 0. 15, the value of 6 $L_{6\sigma}$ is obtained from Table– C for n = 5 as 2.16 and the Six Sigma based control limits for X-bar chart.

$$UCL = \bar{\bar{X}} + L_{6\sigma}\sigma_{6\sigma} = 11.70 + (2.16)(0.15) = 12.02$$

$$CL = \bar{\bar{X}} = 11.70$$

$$LCL = \bar{\bar{X}} - L_{6\sigma}\sigma_{6\sigma} = 11.70 - (2.16)(0.15) = 11.38$$

Table2. Control limits of control charts based on scale estimators

	Three sigma	MAD	Six sigma
LCL	12.36	12.23	12.02
CL	11.70	11.70	11.70
UCL	11.06	11.19	11.38

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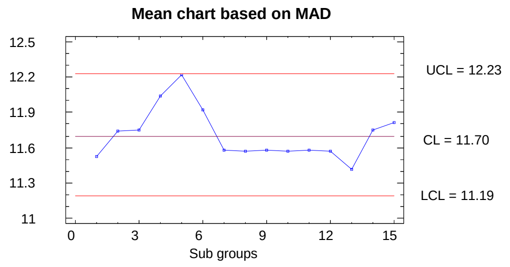
Out of control	0	0	2



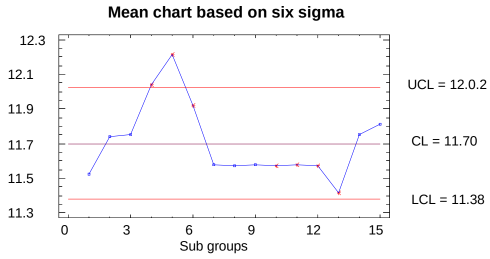


Figure 2: X-Bar chart based on MAD

Figure 3: X-Bar chart based on six sigma

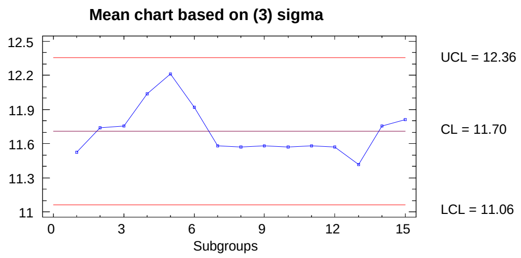


Figure 4: X-Bar chart based on three sigma

II. CONCLUSION

When we compare robust methods, six sigma method have better property than the other robust methods, because control chart based on six sigma have the smallest control interval and also have the most number of out of control point

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الملخص

إن الهدف الرئيسي لأية عملية إنتاجية هي الحصول على مواد ذات جودة عالية ومطابقة للمواصفات، بحيث تلبي رغبات المستهلك. الجودة هي الأساس التي قد يعتقد البعض بأنها جديدة، لكنها قديمة قدم الإنسانية، لأن الإنسان كان ولا يزال في بحث دائم عن شيء جيد ويولي إهتمامه ب الجودة. الجودة تحظى باهتمام كبير في الدول المتقدمة صناعيا كونها مصدرة الاقتصادات وتسعى للتحكم بالأسواق الخارجية.

إن مقياس المقدرات مهم للغاية في العديد من التطبيقات الإحصائية . فمن أكثر مقايس المقدرات شيوعًا هو الانحراف المعياري للعينة. يعتبر الانحراف المعيإري مقياسا غيرحصينا للانحرافات الطفيفة عن التوزيع الطبيعي.

يعرض الدراسة الحالية مقارنة بين مقياس حصين آخر للتشتت وهو (سيغما ثلاثة، سيغما ستة والانحراف المطلق الوسيطي) معتمدا على لوحة سيطرة الوسط الحسابي لشيوارت . عندما نقارن الطرق الحصينة، فإن طريقة ستة سيغما لها خاصية أفضل من الطرق الحصينة الأخرى، لأن لوحة السيطرة المعتمدة على ستة سيغما تمتلك أصغر فترة سيطرة ويحتوي أيضًا على أكبر عدد من نقاط خارج السيطرة.

پوخته

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ئامانجی سەرەکی بۆ هەر پرۆسەیەکی بەرهەم هێنان بریتیە لە گەیشتن بە مەوادێك جۆرەکەی باش بێت وستاندارد بێت وە لەگەڵ داواکاری کریار بگونجێت . جۆری بریتیە لە بنچینەی باشی کە ئەمەش ئارەزووی مرۆڤەکانە هەر لە کۆنەوە چونکە مرۆڤ بەردەوام دەگەرێت بەدوای شتی باش وە گرنگی بە جۆری شتەکان دەدات . وە جۆری جێی بایەخی زۆربە وولاٌتانی پێشکەوتووە پیشەسازیەکانە چونکە سەرچاوەی ئابووریە و هاوکات بۆدەست گرتنە سەر بازارەکانی دەرەوە، خەملاٌندنی خەملێنراوەکان زۆر گرنگە بەتایبەت لە جێبەجیکراوە ئاماریەکان وە گرنگ ترین پێوەرەکانی پەرشوو بلاٌوی بریتیە لە لادانی پێوانەی وە ئەم خەملێنەرە بەهێز نیە لە دیاری کردنی لادانە بچووکەکان لە لەباری ئاسایدا . ئەم تویژینەوە هەڵدەستێت بەبەراورد کردن لەنێوان ئەم خەملێنەرانە (سیٚ سیگما و شەش سیگما و لادانی ناوەراستی رووت) لەسەر بنەمای تابلوٚی شیوارت بوٚ کوٚنتروٚلی جوٚری . گەیشتینە ئەو ئەنجامەی کە خەملێنەری شەش سیگما باشترە لە خەڵێنەرەکانی تر لەبەرئەوەی ئەو تابلۆیەی کە پشتدەبەستێت بە شەش سیگما ماوەی نێوان بەرزترین ئاست و نزمترین ئاست لە تابلوٚی شەش سیگما کەمترە بەراورد بە تابلوٚکانی تر وە هەروەها زۆرترین خاڵ کەوتووەتە دەرەوەی کوٚنتروٚڵ لە تابلوٚی شەش سیگما.

<u>APPENDIX</u>

The Control limits factors for X- Bar chart based MAD		$\sigma_{6\sigma}$ Values for a specified Cp and TL		$L_{6\sigma}$ Values for a Specified Subgroup Size (n)	
n	A_5	CP	TL=1.39	n	$L_{6\sigma}$
2	3.58	1	0.2363	2	3.4160
		1.1	0.2085		
3	4.48	1.2	0.1807	3	2.7891
		1.3	0.1946		
4	4.08	1.4	0.1668	4	2.4155
		1.5	0.1529		
5	3.62	1.6	0.1390	5	2.1604
		1.7	0.1390		
6	3.60	1.8	0.1251	6	1.9722
		1.9	0.1251		
7	3.42	2	0.1112	7	1.8259
		2.1	0.1112		
8	3.38	2.2	0.1112	8	1.7080
		2.3	0.0973		
9	3.32	2.4	0.0973	9	1.6103
		2.5	0.0973		
10	3.26			10	1.5276