



Construction of New Coefficient of Variation Control Chart based on Median Absolute Deviation

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ABSTRACT

Scale estimators are very important in many statistical applications. A robust estimator is an estimator which is insensitive to changes in the underlying distribution and also resistant against the presence of outliers. Several authors have worked on robust control charts and several robust measures are experimented in process control.

Monitoring the coefficient of variation (C.V) is a successful approach to Statistical Process Control when the process mean and standard deviation are not constant. In recent years the (C.V) has been investigated by many researchers as the monitored statistic for several control charts. The aim of this paper is to construct quality control charts with an application and using Coefficient of Variation

Control Chart based on Median Absolute Deviation and comparative with Coefficient of Variation Control Chart based on Standard Deviation. It is concluded that in this study, the process was under control for both cases (phase .1 and phase .2), because all points were fallen within the control limits, which means the product meets the required specifications in (phase-1). We also found that the robust Coefficient of Variation control charts based on Median Absolute Deviation can be used as an alternative instead of Coefficient of Variation Control Chart based on (S.D).

Keywords: quality control chart, coefficient of variation, standard deviation, median absolute deviation.

Introduction

A control chart is a statistical device used for the study and control of a repetitive process. In 1931, Shewhart suggested control charts based on (3) sigma limits. Specifying the control limits is the most important step in designing a control chart. When the limits are narrow, the risk of a point falling beyond the limits increases, and hence increase the false indication that the process is out of control. If the limits are wider, the risk increases the points falling within the limits, falsely indicates that the process is in control (Abu-Shawiesh, 2008). There are many measures of scale available in literature and many control charts are available to control process dispersion or the dispersion of quality characteristics.

Quality Control Charts

A quality control chart (also called process chart) is a graph that shows average for the data (output) or the product fall within the common or normal range of variation if the process is under statistical control. Quality control charts were first

invented by Walter A. Shewhart, and developed by him and his associate. Shewhart's idea was whether the production process is going well and naturally and the points plotted on the chart follow a normal distribution. For these reasons, Shewhart resorted to use the normal distribution in the construction of his charts (Besterfield, 2004). Shewhart control charts consist of three parallel lines which are (Montgomery, 2001):

1. Center Line (or target line) of the control chart is the mean, or overall average, of the quality characteristic that is being measured, and symbolized as CL.
2. The upper control limit (UCL) is the maximum acceptable variation from the mean for a process that is in a state of control.

Mathematically expressed as: $UCL = T + 3\sigma$

3. Lower control limit (LCL) is the minimum acceptable variation from the mean for a process that is in a state of control.

Mathematically expressed as: $LCL = T - 3\sigma$

Classification of control charts

Control charts may be classified into two main types (Besterfield, 2004):

- **Attribute control charts**
- **Variable Control Charts**

The most important types of variable control chart (Montgomery, 2001):

- a. Average – Chart (\bar{x} - chart)
- b. Standard Deviation(S.D)Chart (S-chart)
- c. Individual – Chart(X- chart).

I- Coefficient of variation (C.V) chart based on Median Absolute Deviation(MAD)

The Median Absolute Deviation from the sample median (MAD) is a very robust scale estimator than the sample standard deviation (Rousseeuw, 1993). It measures the deviation of the data from the sample median. The MAD is often used as an initial value for the computation of more efficient robust estimators. Let

x_1, x_2, \dots, x_n be a simple random sample of size n observation taken over m subgroups, and then the MAD is defined as (Abu-Shawiesh, 2008):

$$MAD = 1.4826 * MD \{|X_i - MD|\} \quad \dots(1)$$

Where, MD is the median of $X_1, X_2 \dots X_n$. The average of the MAD is computed using:

$$\overline{MAD} = \sum_{i=1}^m MAD / m \quad \dots (2)$$

When MAD is used as an estimate of variability, then MAD will be used as a replacement of the standard deviation (S), thus, $\hat{\sigma} = b_n \overline{MAD}$ (Abu-Shawiesh, 2008). The proposed robust control chart based on the MAD estimator is a chart of subgroup standard deviations in which the control limits for the sake of robustness are set using the median absolute deviation from the sample median (MAD). Thus we may set the control limits and Central lines for the based on the MAD are calculated as follows (Adekeye, 2012):

$$\begin{aligned} UCL_{MAD} &= C_4 \hat{\sigma} + 3\hat{\sigma}\sqrt{1 - C_4} \\ &= C_4 b_n \overline{MAD} + 3b_n \overline{MAD}\sqrt{1 - C_4} \\ &= B_6 b_n \overline{MAD} \\ &= B_6^* \overline{MAD} \end{aligned} \quad \dots (3)$$

$$CL_{MAD} = C_4 \hat{\sigma} = C_4 b_n \overline{MAD} = C_4^* \overline{MAD} \quad \dots (4)$$

$$\begin{aligned} LCL_{MAD} &= C_4 \hat{\sigma} - 3\hat{\sigma}\sqrt{1 - C_4} \\ &= C_4 b_n \overline{MAD} - 3b_n \overline{MAD}\sqrt{1 - C_4} \\ &= B_5^* \overline{MAD} \end{aligned} \quad \dots (5)$$

$$C_4 = \frac{E(\overline{S})}{\hat{\sigma}} = C_4^* b_n \quad \dots (6)$$

$$B_5^* = B_5 b_n = C_4 - 3\sqrt{1 - C_4^2} \quad \dots (7)$$

$$B_6^* = B_6 b_n = C_4 + 3\sqrt{1 - C_4^2}$$

Thus we may set the control limits and central line for the %CV chart based on MAD, for specified limits the calculations are:

$$\%C.V(CenterLine) = \frac{C^* \overline{MAD}}{\overline{\bar{X}}} * 100 \quad \dots (8)$$

Where MAD is the specified centerline of the data displayed on the MAD chart and ($\overline{\bar{X}}$) is the specified mean as defined in the control limit record.

$$\%CV(UCL) = \frac{UCL_{MAD}}{\overline{\bar{X}}} * 100 \quad \dots (9)$$

$$\%CV(LCL) = \frac{LCL_{MAD}}{\overline{\bar{X}}} * 100$$

Where UCL_{MAD} and LCL_{MAD} are the specified control limits from the MAD chart and ($\overline{\bar{X}}$) is the process mean as defined in the control limit record.

%C.V is the percentage of the mean represented by the standard deviation a relative measure of variation.

$$\%C.V(plot\ point) = \frac{S}{\bar{X}} * 100 \quad \dots (10)$$

Where (\bar{X}) is the subgroup average and (S) the subgroup standard deviation.

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{X})^2}{n - 1}} \quad \dots (11)$$

(B^*_5, B^*_6, b_n, C^*_4): Factors for Computing Central Lines and Control Limits based on sample size (Abu-Shawiesh, 2008) (Montgomery, 2001).

II- Coefficient of variation (C.V) chart based on Standard Deviation (S.D)

The centerline and control limits on the %CV chart are calculated based on (S) chart. For specified control limits the calculations are (Hassani, 2009):

$$(CenterLine)\%C.V = \frac{\bar{S}}{\overline{\bar{X}}} * 100$$

$$(UCL)\%C.V = \frac{UCL_S}{\bar{\bar{X}}} * 100$$

$$(LCL)\%C.V = \frac{LCL_S}{\bar{\bar{X}}} * 100$$

Then the control limits and central line for the (S.D) control chart are calculated as follows:

$$UCL_S = \bar{S} + 3\sigma_S \quad \dots (12)$$

$$CL_S = \bar{S} \quad \dots (13)$$

$$LCL_S = \bar{S} - 3\sigma_S \quad \dots (14)$$

Table (1): Factors for Computing Central Lines and Control Limits

OBSERVATIONS INSAMPLE, n	CHART FOR STANDARD DEVIATIONS			
	FACTOR FOR CENTRAL LINE		FACTORS FORCONTROL LIMITS	
	C_4^*	b_n	B_5^*	B_6^*
2	0.954	1.196	0	3.117
3	1.325	1.495	0	3.403
4	1.256	1.363	0	2.846
5	1.134	1.206	0	2.369
6	1.142	1.200	0.025	2.249

APPLICATION PART:

This section deals with the use (Phase.1and Phase.2) for construction and application of New Coefficient of Variation Control Chart based on Median Absolute Deviation Coefficient. We collected the data from the factory (Coca-Cola /Erbil) and the data representing the quality properties of drink (750 ml) for Coca-Cola product.

Phase.1 // Construction of Chart

In order to build quality control charts, we used (104) observations drink (750 ml) for Coca-Cola product, and divided into (26) samples and each sample consisting of (4) observations as shown in table (2),

Table (2): data representing the quality properties of drink (750 ml) for Coca-Cola product

No. samples	x1	x2	x3	x4	X-bar	S.D	%C.V	MAD	No. samples	x1	x2	x3	x4	X-bar	S.D	%C.V	MAD
1	750.36	750.14	750.36	751.36	750.555	0.5466	0.07283	0.163086	14	750.44	750.32	750.68	751.78	750.805	0.66701	0.08884	0.266868
2	750.78	750.86	751.86	751.96	751.365	0.63148	0.08404	0.800604	15	751.7	750.6	751.28	750.62	751.05	0.53628	0.0714	0.504084
3	751.12	751.38	751.28	751.22	751.25	0.10893	0.0145	0.118608	16	750.62	750.26	750.52	750.3	750.425	0.17311	0.02307	0.192738
4	750.28	751.02	750.88	750.36	750.635	0.36964	0.04924	0.44478	17	750.96	750.24	750.84	751.58	750.905	0.54927	0.07315	0.533736
5	750.48	750.5	750.56	750.48	750.505	0.03786	0.00504	0.014826	18	750.16	750.98	750.92	750.52	750.645	0.3824	0.05094	0.340998
6	750.08	750.2	750.2	751.04	750.38	0.44362	0.05912	0.088956	29	750.54	750.62	750.8	751	750.74	0.20461	0.02725	0.192738
7	750.7	750.86	750.42	750.4	750.595	0.22353	0.02978	0.22239	20	752.36	752.18	750.69	751.4	751.658	0.76787	0.10216	0.711648
8	750.2	750.02	750.2	749.56	749.995	0.30216	0.04029	0.133434	21	752.16	751.32	750.9	750.32	751.175	0.77414	0.10306	0.7413
9	750.49	750.7	751.02	750.34	750.638	0.29466	0.03925	0.266868	22	750.8	750.8	751.08	751.44	751.03	0.30353	0.04042	0.207564
10	750.5	751.6	750.54	750.66	750.825	0.52112	0.06941	0.118608	23	750.22	750.28	750.12	750.56	750.295	0.18859	0.02514	0.118608
11	750.66	750.52	750.62	750.9	750.675	0.16114	0.02147	0.103782	24	750.26	750.4	750.54	750.44	750.41	0.11605	0.01546	0.103782
12	750.9	750.16	750.42	750.74	750.555	0.3304	0.04402	0.355824	25	750.5	750.44	750.5	750.56	750.5	0.04899	0.00653	0.044478
13	750.34	750.6	750.48	750.54	750.49	0.11136	0.01484	0.088956	26	750.92	750.14	750.54	750.48	750.52	0.31958	0.04258	0.29652

Construction of control limits and center line for (MAD) chart

$$UCL_{MAD} = B_6^* \overline{MAD} = (2.846)(0.27599) = 0.7854$$

$$CL_{MAD} = C_4^* \overline{MAD} = 1.236 * 0.27599 = 0.3411$$

$$LCL_{MAD} = B_5^* \overline{MAD} = (0)(0.27599) = 0$$

Construction of new control limits and center line for (%CV) chart based on MAD

$$\%CV(UCL) = \frac{UCL_{MAD}}{\overline{X}} * 100 = \frac{0.7854}{750.7162} * 100 = 0.104$$

$$\%CV(CL) = \frac{C_4^* \overline{MAD}}{\overline{X}} * 100 = \frac{0.3411}{750.7162} * 100 = 0.045$$

$$\%CV(LCL) = \frac{LCL_{MAD}}{\bar{\bar{X}}} * 100 = \frac{0}{750.7162} * 100 = 0$$

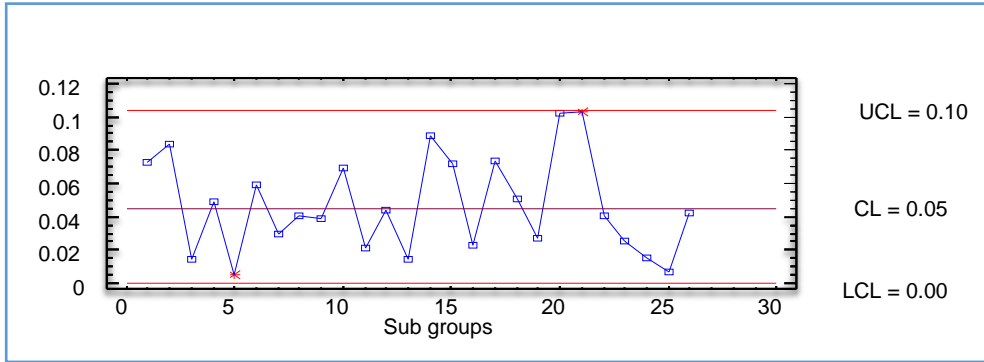


Figure 1: Coefficient of variation (C.V) chart based on Median Absolute Deviation

Figure (1) shows that all the points are fallen within the limits of control. This means that the above chart can be relied upon and used in the future for the same properties of quality from which we obtained the data for the purpose of control and monitoring of future production.

Construction of control limits and center line for Classical (S.D) chart

$$UCL_s = \bar{S} + 3\sigma_s = (2.846)(0.27599) = 0.99$$

$$CL_s = \bar{S} = 1.236 * 0.27599 = 0.05$$

$$LCL_s = \bar{S} - 3\sigma_s = (0)(0.27599) = 0$$

Construction of new control limits and center line for (%CV) chart based on (S.D)

$$\%CV(UCL) = \frac{UCL_s}{\bar{\bar{X}}} * 100 = \frac{0.99}{750.7162} * 100 = 0.1318$$

$$\%CV(CL) = \frac{\bar{S}}{\bar{\bar{X}}} * 100 = \frac{0.4}{750.7162} * 100 = 0.05$$

$$\%CV(LCL) = \frac{LCL_s}{\bar{\bar{X}}} * 100 = \frac{0}{750.7162} * 100 = 0$$

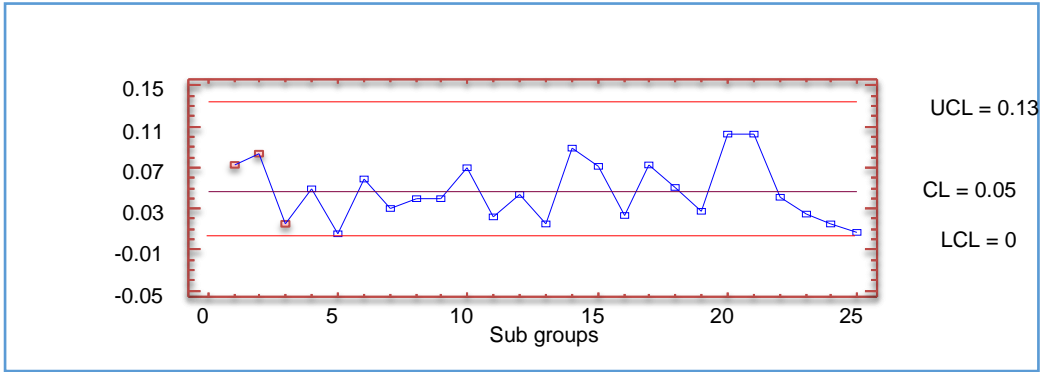


Figure 2: Coefficient of variation (C.V) chart based on (S.D)

Phase.2 // Practical Application of Chart

In this part, we use the above charts for the purpose of controlling and monitoring the mean for (80) observations (drink (750 ml) for Coca-Cola) as given in the table (3).

Table (3): data representing the quality properties of drink (750 ml) for Coca-Cola product

No. samples	x1	x2	x3	x4	%C.V	No. samples	x1	x2	x3	x4	C.V%
1	751.02	750.8	751.08	751.44	0.03535	11	750.96	750.24	750.84	751.58	0.07315
2	750.54	750.42	750.44	750.56	0.00936	12	750.16	750.6	750.6	750.52	0.02799
3	750.49	750.92	750.14	750.34	0.04411	13	750.54	750.62	750.8	751	0.02725
4	750.5	751.6	750.54	750.66	0.06941	14	752.36	752.18	750.69	751.4	0.10216
5	750.66	750.52	750.62	750.9	0.02147	15	750.2	750.44	750.32	750.68	0.02731
6	750.9	750.16	750.42	750.74	0.04402	16	751.02	750.7	750.86	750.42	0.03408
7	750.34	750.6	750.48	750.54	0.01484	17	750.54	750.2	750.02	750.2	0.02896
8	750.44	750.32	750.68	751.78	0.08884	18	750.62	750.96	750.24	750.84	0.04215
9	750.7	750.86	750.42	750.4	0.02978	19	750.42	750.16	750.6	750.6	0.02773
10	750.2	750.02	750.2	749.56	0.04029	20	750.2	750.54	750.62	750.8	0.0335

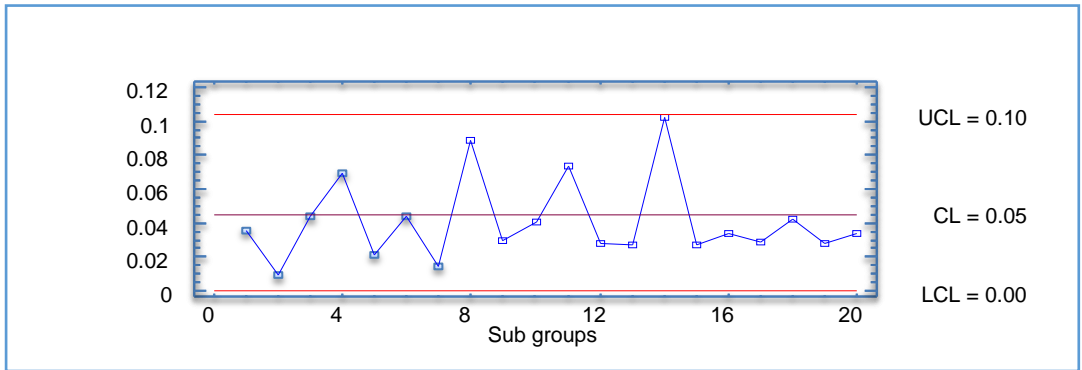


Figure 3: Practical application of (C.V) chart based on Median Absolute Deviation

From figure (3), we notice that all points are located within the control limits and this means the process is statistically under controls.

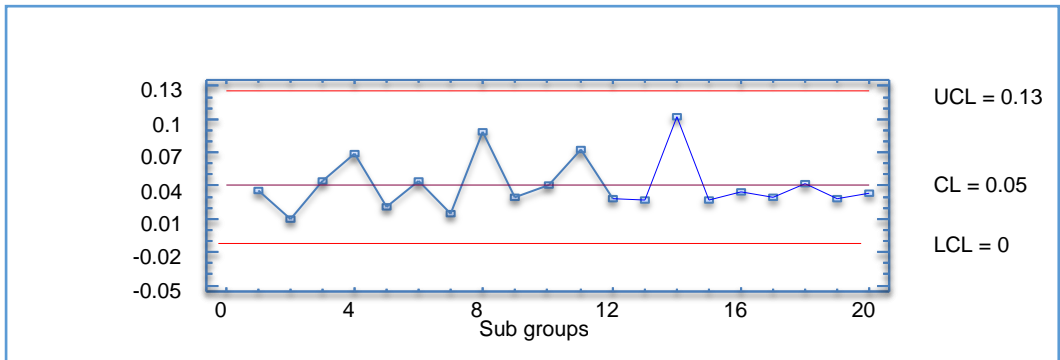


Figure 4: Practical application of (C.V) chart based on (S.D)

From figure (4), we notice that all points are located within the control limits

CONCLUSION

To sum up, in this research paper we found the following:

1. The distance between the upper and lower limits in the control chart based on Median Absolute Deviation is less than the control chart based on standard Deviation. This means that the Median Absolute Deviation is an important estimate of reducing the size of the error go so far as to.
2. Obtaining an appropriate data for the construction (C.V) chart based on Median Absolute Deviation for the first time (phase-1) for factory (coca – cola/Erbil) to control and monitoring of future production of drink (750 ml)

for Coca-Cola. This means that the product meet the required specifications in (phase-1).

3. Through (phase-2) we observed that the process is under control, because all points are located within the control limits.
4. The robust Coefficient of Variation control charts based on Median Absolute Deviation can be used as an alternative method instead of Coefficient of Variation Control Chart based on (S.D).

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پوخته

پیشبینی کراوه کانی پیوه گرنگن له زۆریه جی به جی کردنه ئاماریه کان. و پیشبینی به هیژ بریتیه له پیشبینی وورد بۆگوراوکان له دابهش بووی بنه رتی و ههروهها بریتیه له پیشبینی به رهنگار بوو له کاتی بوونی به های په رگر یان نامۆ. وه زۆر له دانهره کان ئیشیان کردوو له سهه تابلوی کۆنترۆلی جۆری به هیژ وه پیوه ره فره به هیژه کان تاقی کراونه ته وه له پرۆسه ی کۆنترۆل کردن.

چاودیری کردنی هاوکۆلکه ی په رش و بلاوی (C.V) بریتیه له رپرۆیکی سهه رکه وتوووه بۆکۆنترۆلی ئاماری له پرۆسه کان کاتیک نه گه ر هاتوو تیکراو لادنی پیوانه ی پرۆسه که گوراو بیت. وه له م سالانه ی کۆتایی هاوکۆلکه ی په رش و بلاوی (C.V) تاقی کرایه وه له لایان توێژه ره کانی ئامار بۆ چاودیری کردن له تابلویه کانی کۆنترۆلی جۆری. ئامانج له م توێژینه وه بریتیه له دروست کردنی تابلوی کۆنترۆلی جۆری له گه ل جی به جی کردن. سهه رپایی به کارهینانی هاوکۆلکه ی په رش و بلاوی به پشت به ستن به لادانی رووت بۆ پیوه ری ناوه راست. له ده رنه نجامی نه م توێژینه وه یه ده رکه وت که پرۆسه که له ژیر کونترۆله له هه ردوو باردا (باری ۱ و باری ۲) له به رنه وه ی هه موو خاله کان ده که وپته نیوان هه ردوو ئاستی کۆنترۆل. نه مه ش مانای نه وه یه به ره مه که هاوشپوه ی تابه تمه ندی داواکراوه له (باری ۱). وه هه ره وه تابلوی کۆنترۆل بۆ هاوکۆلکه ی په رش و بلاوی به پشت به ستن به لادانی رووت بۆ پیوه ری ناوه راست جیگه وه یه بۆ تابلوی کۆنترۆل بۆ هاوکۆلکه ی په رش و بلاوی به پشت به ستن به لادانی پیوانه ی (S.D).

مستخلص

يعتبر مقدرات القياس مهمة جدا في العديد من التطبيقات الإحصائية. والمقدر الحصين هو مقدر حساس للتغيرات في التوزيع الأساسي وأيضا هو مقدر مقاوم في حال وجود القيم المتطرفة او الشاذة. لقد عمل العديد من المؤلفين على لوحات السيطرة الحصينة ومقاييس حصينة متعددة قد أختبرت في عملية السيطرة.

إن مراقبة معامل الاختلاف (C.V) يعتبر نهجا ناجحا للسيطرة الإحصائية على العمليات عندما يكون المعدل والانحراف المعياري للعملية ليس ثابتا. ففي السنوات الأخيرة تم فحص معامل الاختلاف (C.V) من قبل العديد من الباحثين كإحصاءة مراقبة للعديد من لوحات السيطرة. الهدف من هذا البحث هو تكوين لوحة السيطرة مع تطبيق، إضافة الى استخدام معامل الاختلاف معتمدا على الانحراف المطلق للوسيط. وقد تم الإستنتاج في هذه الدراسة أن العملية تحت السيطرة لكلا الحالتين (المرحلة ۱ والمرحلة ۲) لأن جميع النقاط وقعت ضمن حدود السيطرة، مما يعني أن المنتج مطابق المواصفات المطلوبة في (المرحلة ۱). ووجدنا أيضا أن لوحة السيطرة باستخدام معامل الإختلاف الحصين معتمدا على الانحراف المطلق للوسيط يمكن أن يستخدم كبديل لوحة السيطرة باستخدام معامل الإختلاف معتمدا على الانحراف المعياري (S.D)