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New Weights in Laplacian Smoothing on Triangular Mesh

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ABSTRACT

Mesh smoothing is one of the basic procedures for improvement of mesh quality. Most smoothing techniques move vertices of the mesh without changing topology of the connectivity. Laplace smoothing is one of the simplest and efficient algorithm, where in each step vertex of the mesh is move to the barycenter of its neighbors. The only problem with Laplacian smoothing is surface shrinking when it is performed iteratively. In this paper, three relatively simple weights proposed in Laplacian smoothing which has less surface shrinking from the previous weights. The performance and effectiveness of the presented weights are demonstrated on two knowing simply and doubly connected geometrical shapes respectively.

1. INTRODUCTION

The polygonal discretization, particularly triangular discretization are widely used for representing discrete surface in computer applications for example smooth shading [1, 2], surface reconstruction from cloud of points and in numerical simulation methods such boundary and finite element methods. More applications can be found in direct rendering of points in computer graphics [3]. This discretization leads undesirable noise and small-scale oscillation. Therefore, the quality improvement for the discrete surface is needed. The mesh improvement has been widely used in many areas, such as mesh generation [4–6], simplifying discretization [7], dynamic grid or mesh deformation [8–9], and other mesh processing procedures. Mesh improvement approach can be basically divided into two main categories, topological optimization and smoothing (also called geometrical optimization). Topological optimization changes the topology of a mesh, that is, the node-element connectivity relationship, while smoothing or geometrical optimization improves mesh quality by simply moving or adjusting node positions without changing the topology of mesh. This paper will focus on the latter, smoothing [10].

The most two popular approaches for smoothing and reducing the noises in triangular surface are minimizing the total energy stored in the line segments that connects the points in the mesh and Laplacian smoothing [11].

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2. PROBLEM STATEMENT

Laplacian smoothing is a common, simple and efficient technique to improve of the polygonal mesh in general and particularly in the triangular mesh. Most smoothing techniques including Laplacian smoothing move the vertices of the polygonal mesh without changing the connectivity of the faces and vertices. The smoothed mesh has exactly the same number of vertices and faces as the original (see figure 1). Laplacian smoothing is an iterative process, where in each step every vertex of the mesh is moved to the barycenter of its neighbors. A great deal of mesh smoothing algorithms have been proposed in the literature. The most common techniques are based on Laplace smoothing [12, 13]. Taubin [12] introduced signal processing on surfaces that is based on the definition of the Laplacian operator on meshes and developed a fast and simple iterative Laplacian smoothing scheme. When Laplacian smoothing is applied to a noisy three dimensional triangular mesh without constrain, noise is removed but the problem is that this procedure produce Shrinking.

2.1. LAPLACIAN SMOOTHING

Suppose a surface S is discretized into triangular elements and composed of a set of n vertices $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_n$ and a set of m faces $f_1, f_2, \ldots, f_n$. The umbrella operator $\boldsymbol{U}$ for $\boldsymbol{p}$ is defined by;

$$\boldsymbol{U}(\boldsymbol{p}) = \frac{\sum_i w_i \boldsymbol{r}_i}{\sum_i w_i} - \boldsymbol{p}, \tag{1}$$

where summation is taken over all neighbors of $\boldsymbol{p}$, w_i are positive weights. The Laplacian smoothing for any point $\boldsymbol{p}$ is;

$$\boldsymbol{p}^{new} = \boldsymbol{p}^{old} - \lambda \boldsymbol{U}(\boldsymbol{p}^{old}), \tag{2}$$

where $0 < \lambda < 1$ is the weights. The geometric idea behind Laplacian smoothing algorithm (2) is to reduce the high noise surface and tends to flatten every patch of the surface as shown in figure 1.

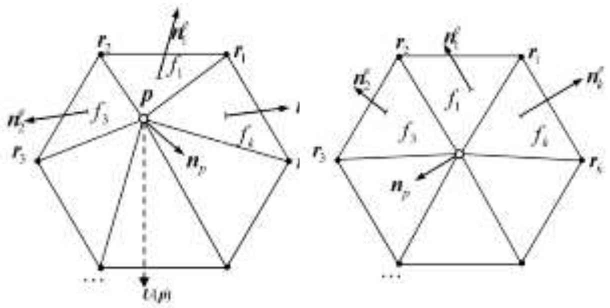

FIGURE 1. Sketch of the surrounding triangular faces to the arbitrary node $\boldsymbol{p}$.

The weights w_i can be chosen in different ways. The simplest chose is;

$$w_i = 1. \tag{3}$$

According to [14] the weights to the inverse distance between $\boldsymbol{p}$ and its neighbors produce good results.

$$w_i = \|\boldsymbol{p} - \boldsymbol{r}_i\|^{-1}. \tag{4}$$

Note that the Laplacian smoothing in (2) is applied repeatedly.

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3. NEW WEIGHTS SCHEMES w_i

In this section, we suggest three new weight factors w_i, to reduce the shrinking effects which occurs during the Laplacian smoothing. Suppose a typical node $\boldsymbol{p}$ on the mesh, and $\boldsymbol{r}_i$ (i=1, 2, …, M_i) are neighbor nodes $\boldsymbol{p}$. First, we introduce the angle between unit normal of $\boldsymbol{p}$ and $\boldsymbol{r}_i$ as w_i,

$$w_i = \arccos(\boldsymbol{n}_p . \boldsymbol{n}_{r_i}), \tag{5}$$

where the unit norm arbitrary node $\boldsymbol{n}$ is calculated by averaging the normal of the surrounded planar triangular elements,

$$\boldsymbol{n} = \frac{\sum_{i=1}^{k} A_i \boldsymbol{n}_i^e}{\left\| \sum_{i=1}^{k} A_i \boldsymbol{n}_i^e \right\|}, \tag{6}$$

Where $\boldsymbol{n}_i^e$ is the unit normal of the face f_i and A_i is the area of the triangle f_i (see figure 1)
The second weight method w_i is the sum of all length of the edges that passes the node $\boldsymbol{Q}_i$,

$$w_i = \sum_{j}^{M_i} \left\| \boldsymbol{Q}_i - \boldsymbol{r}_j \right\|, \tag{7}$$

where $\boldsymbol{r}_j$, M_i are the surrounded nodes and the number of surrounded of Q_i, respectively.
Third weigh factor is,

$$w_i = v_i \tag{8}$$

where v_i is the volume of the pyramid that generated by projecting the triangle f_i on the plane Z=0. Therefore, if we denote the vertices of the triangle f_i in the local coordinates by $\boldsymbol{r}_i', \boldsymbol{r}_{i+1}', O$, then the base vertices of the pyramid are $\boldsymbol{r}_i', \boldsymbol{r}_{i+1}', \boldsymbol{r}_i'', \boldsymbol{r}_{i+1}''$, where $\boldsymbol{r}_i'', \boldsymbol{r}_{i+1}''$ are projected points of $\boldsymbol{r}_i', \boldsymbol{r}_{i+1}'$, on the plane Z=0, respectively (see figure 2b).

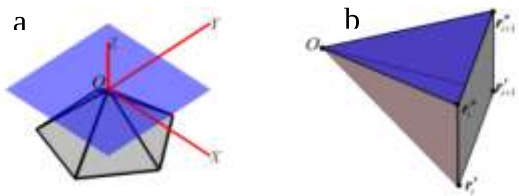

FIGURE 2. (a) Sketch of the surrounding triangular elements to the node p at the local coordinates system *O-XYZ*. (b) Pyramid with origin *O* is apex and base vertices are $\boldsymbol{r}_i', \boldsymbol{r}_{i+1}', \boldsymbol{r}_i'', \boldsymbol{r}_{i+1}''$.

4. NUMERICAL EXAMPLES

Two geometrical shapes are destroyed randomly in this section, then the algorithm for smoothing sections 2 and 3 are implemented. In the first one, suppose a unit sphere is discretized into optimal triangular surface. Then it is randomly destroyed in all directions (figure 3 (a)). In figure 3(b)-(f) the weights in equations (3)-(5), (7)-(8) are used in the Laplacian smoothing (2), with λ=0.5 and 5 time repeated respectively, here the number of triangle is 2000.

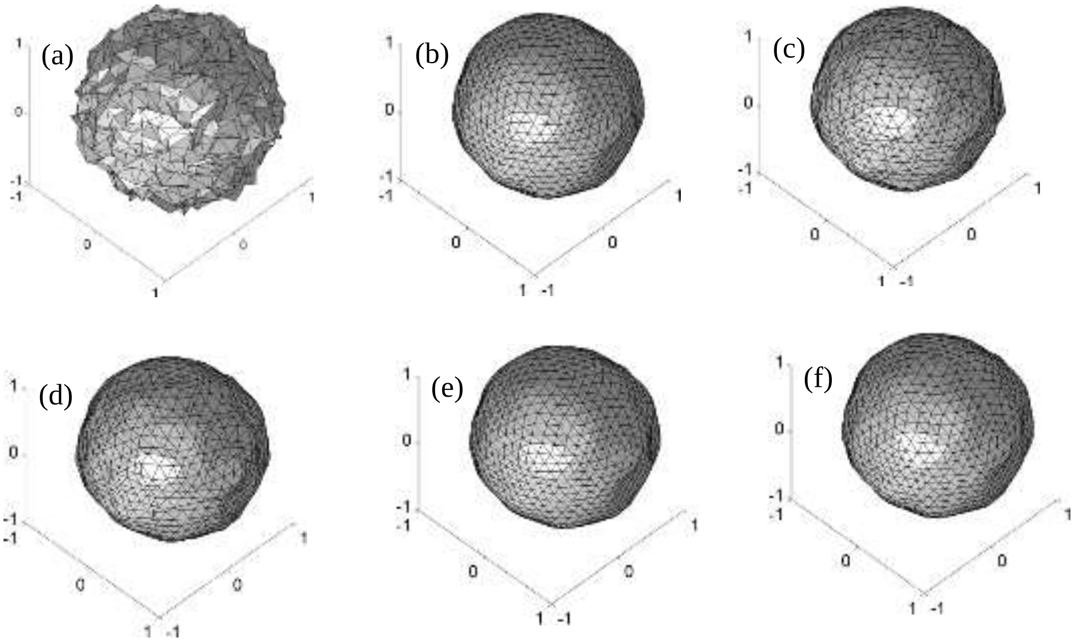

FIGURE 3. The unit sphere surface with 980 triangular elements (a) Deformed the unit sphere randomly in all directions. Laplacian smoothing applied 5 times, λ=0.5 and with using weight functions (b) in equation (3); (c) in equation (4); (d) in equation(5); (e) in equation(7) and (f) in equation(8).

In Figure 4 the same process of figure 2 repeated with the increasing number of iterations into 20. It is clear that the surfaces are smoother in the previous cases. In the table 1, relative changes in the volume have been presented, when the Laplacian smoothing applied with all weights in sections 2-3. The same parameters in the case figure 3 used but for different number of triangles. It is clear that the relative change of volume reduce with increasing triangle number (*m*) on the surface and the volume changes smaller in the proposed weights.

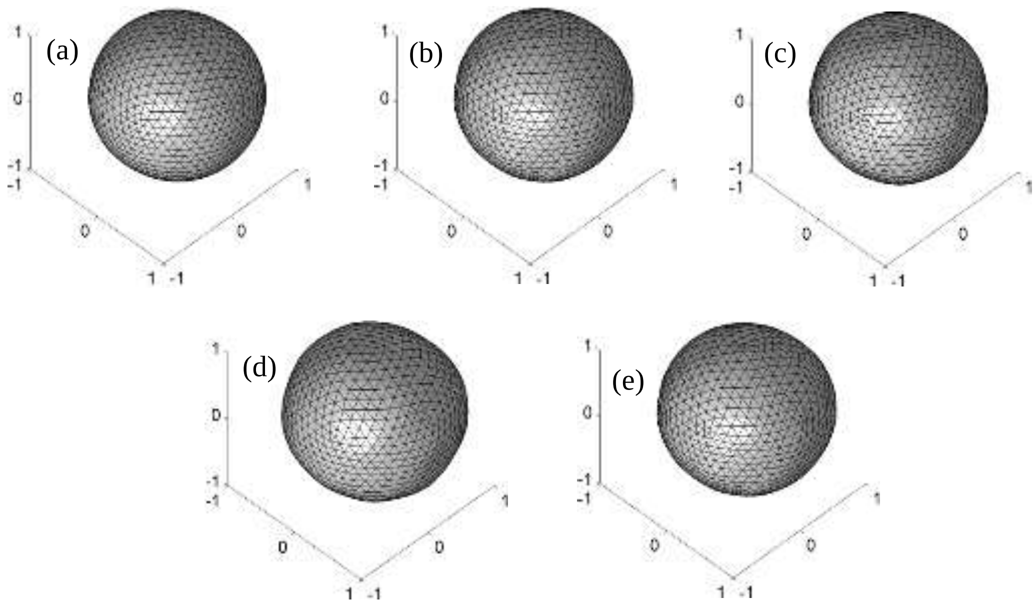

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FIGURE 4. Laplacian smoothing applied 20 times, λ=0.5 and with using weight functions (a) in equation (3); (b) in equation (4); (c) in equation(5); (d) in equation(7) and (e) in equation(8).

TABLE 1. Relative change of Volume ΔV when Laplacian smoothing implemented 20 times with all weights in sections 2-3 for difference number of elements m.

m	ΔV %				
	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (7)	Eq. (8)
500	0.6	0.54	0.54	0.3	0.28
980	0.37	0.35	0.31	0.28	0.28
2000	0.19	0.17	0.15	0.12	0.12
3920	0.1	0.09	0.08	0.07	0.07

The second example is a torus, which given by,

$$\begin{aligned} x(\theta,\varphi) &= (r_1 + r_2\cos\theta)\cos\varphi \\ y(\theta,\varphi) &= (r_1 + r_2\cos\theta)\sin\varphi \qquad 0 \le \theta,\varphi \le 2\pi, \\ z(\theta,\varphi) &= r_2 \sin\theta \end{aligned} \tag{9}$$

where r_1=0.8 and r_2=0.4 as showed in figure 5(a). The points in on the torus can be considered as a parabolic, elliptical or hyperbolic point. Therefore, this type of the surface is good for investigation. Half of the torus perturbed randomly, then all weights in section 2 and 3 in the Laplacian smoothing implemented for 20 cycles wit λ=0.5. The number of elements is 8214.

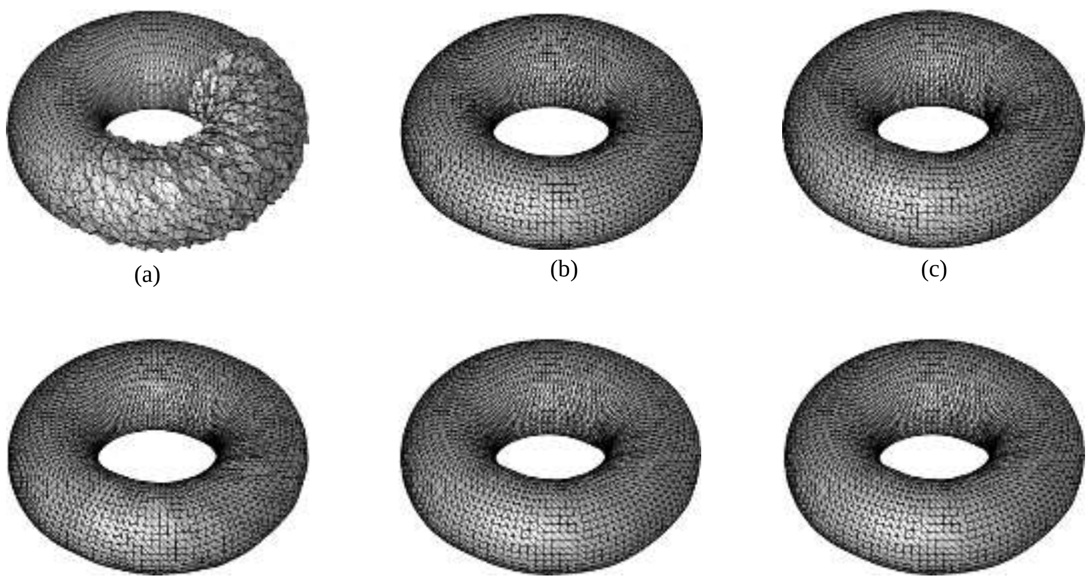

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FIGURE 5. Laplacian smoothing applied 20 times, λ=0.5 and with using weight functions (a) in equation (3); (b) in equation (4); (c) in equation (5); (d) in equation(7) and (e) in equation(8).

TABLE 2. Relative change of Volume ΔV when Laplacian smoothing implemented 20 times with all weights in sections 2-3 for difference number of elements *m*.

m			**ΔV %**		
	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (7)	Eq. (8)
600	0.93	0.93	0.92	0.90	0.70
1014	0.80	0.76	0.72	0.68	0.50
1944	0.58	0.52	0.56	0.52	0.45
4056	0.32	0.28	0.27	0.25	0.21
8214	0.19	0.15	0.13	0.10	0.06

5. CONCLUSION

Laplacian smoothing is a simple and efficient technique to improve mesh quality. The only problem in the Laplacian smoothing is when it is implemented iteratively, the surface shrinking. The weight factor may have rule for the amount of shrinking. In this paper, three new weights proposed and numerical examples are considered for the comparison purpose.

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