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Developing a Bayesian technique employing a posterior based on mode to estimate parameters in multiple linear regression (simulation study)

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ABSTRACT

The process of estimating the parameters of regression is still one of the most important. Despite the large number of papers and studies written on this subject, these studies differ in the techniques followed in the process of estimation, whether they are classic or Bayesian. In this study, we developed a Bayesian technique employing a posterior-based mode to estimate parameters in multiple linear regression. The best multiple linear regression model for the data may be obtained based on the mean squared error after comparing the Bayesian posterior based on mode and the traditional method (ordinary least squares) by combining simulated and real data with a MATLAB program made especially for this purpose. The study finds that, compared to the traditional approach, the Bayesian posterior based on mode approach yields more accurate parameter estimates and In terms of the RMSE statistical criterion, the best results for estimating the multiple linear

regression model.

1: Introduction

Regression analysis is regarded as one of the most essential statistical approaches used by researchers to examine data in their respective domains, such as industry, biology, social science, and manufacturing, in order to achieve the best results (S. A. Obed, D. M. Saleh & D. I. Jamil. (2023)). this problem is answered by developing a correct formula for the link between the many phenomena represented by variables, and these variables are submitted to the regression formula in its various forms. The model of regression Formulas are extremely helpful in determining the direction of the explanatory variables with which we are dealing. It is done by the researcher, who is also aware of the influence range that these variables have on the response. Aside from the interpretation ratio of the regression model's contribution to explaining the link between the response variable and the explanatory variables, all of this is accomplished through the process of estimating the model's parameters (Geweke J. (2005)).

To examine the relationship between the dependent variable and two or more independent variables, multiple linear regression models can be used. Before performing linear regression modeling, it is necessary to establish that each explanatory variable ha a relationship or correlation with the response variable, and that the relationship between the outcome variable and all independent variables is linear. The most common models are simple linear and multiple linear.

The Bayesian technique is one of the strategies that may be used to estimate the parameters of a regression model (Philippe G, Alain D. and Mylene B., (2020)). The distinction between frequentist and Bayesian approaches is the parameter viewpoint. The Bayesian approach regards the parameters as a random variable, rather than a single value, as the frequentist approach does.

In practice, Bayesian procedures are widely utilized and accepted. The Bayes model is based on the computation of posteriors based on defined priors and the likelihood function of data (T. H. Ali & D. M. Salah (2021)). It is critical for researchers to correctly identify priors because incorrect priors might lead to skewed estimates or make posterior computation difficult. In this section, we will go over some of the most popular priors, how posteriors are derived using priors, and how prior selection effects posterior computation. In a Bayesian analysis, we begin with our model, which is the observed data distribution conditioned on its parameters (Goldstein, M., (1976)). This is also known as the likelihood function and is, for the most part, equivalent to the classic likelihood function. To update our model, we construct a distribution for the model's parameter(s), which is based on previous beliefs. This distribution is known as a prior distribution, and the parameters within it are known as hyper parameters.

2: Linear regression

One of the most straightforward and well-liked methods is linear regression. This statistical method is used for predictive analysis. Using linear regression, a continuous, real, or quantitative variable—such as sales, salary, age, or product price—is predicted. The linear regression algorithm gets its name from the fact that a dependent (y) variable and one or more independent (x) variables show a linear relationship. By using linear regression, which demonstrates a linear relationship, the value of the dependent variable is established in relation to the value of the independent variable (D.M. Saleh, D. H. Kadir, and D. I. Jamil (2023)).

3: Multiple Linear regression model

Multiple regression, also referred to as multiple linear regression, is a statistical method for predicting the outcome of a response variable using a number of explanatory variables. Representing the linear relationship between the explanatory (independent) variables and the response (dependent) variables is the goal of multiple linear regression. Since several explanatory variables are involved, multiple regression is essentially an extension of ordinary least-squares (OLS) regression (T. H. Ali & D. M., Salah (2022)).

It can be defined as follows when taking into account the multiple linear regression model with n observations and p independent variables:

$$y_i = \beta_o + \beta_1 x_{1i} + \ldots + \beta_p x_{pi} + \varepsilon_i \quad \ldots. (1)$$

Where, for i=n observations:

y =dependent variable.

x_i=explanatory variables.

β_o=y-intercept (constant term).

β_p=slope coefficients for each explanatory variables.

ε_i=the model's error term (also known as the residuals).

The ordinary least squares (OLS) method is used to estimate the parameters, which seeks to minimize the residual sum of squares, also referred to as the sum of squared variances between the observed and fitted responses(Chib, S., e tal ,(2008)):

$$\sum_{i=1}^{n} e_i^2 = e'e = (Y - X\beta)'(Y - X\beta) \quad \ldots\ldots (2)$$

The least squares estimate of (β) is obtained by minimizing eq. (2).

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad \ldots. (3)$$

$$\hat{\sigma}^2 = \frac{(Y - X\beta)'(Y - X\beta)}{n - p - 1} \quad \ldots.. (4)$$

4: Bayesian Approach

A different concept than that which underpins significance tests and confidence intervals informs the approach to statistics known as Bayesian statistics. The update of evidence is mainly the focus. A prior distribution of the quantities of interest is used in a Bayesian analysis to describe initial uncertainty. The likelihood presents a summary of the current facts and the underlying assumptions. Following that, the prior distribution and likelihood can be combined to get the posterior distribution for the quantities of interest. With point estimates and credible intervals, which resemble conventional estimates and confidence intervals, the posterior distribution can be summarized. In Bayesian statistics, the prior distribution is a contentious topic. Even while ideas about effects can be represented as a prior distribution, it may appear unusual to combine factual trial data with an individual's subjective judgment. Therefore, using non-informative prior distributions to depict a position of prior ignorance is a typical practice in meta-analysis. In particular, the primary comparison demonstrates this. Prior distributions may, however, be applied to additional parameters in a meta-analysis, such as the degree of variation among studies in a random-effects analysis. In some cases, especially when there are few studies included in the meta-analysis, it may be helpful to include judgment or outside evidence. To determine how the outcomes vary from any set assumptions, it is crucial to conduct sensitivity assessments. (Box & Taio, 1992).

4.1: Bayesian Approach for parameters estimation

For the purpose of explaining the Bayesian method of estimation in general terms, after obtaining the subsequent probability density function for the parameters, the Bayesian estimates are the estimated values $\hat{\theta}_{B1}, \hat{\theta}_{B2}, \ldots, \hat{\theta}_{Bp}$ to $\theta_1, \theta_2, \ldots \theta_p$ respectively, which makes the posterior probability density function at its maximum, i.e. finding the mode value of the posterior mode (Chang, T. & Eaves, D. M. (1992)).

5. Methodology:

5.1: posterior mode based on Informative Prior Probability Distribution

We have the following model:

$\underline{Y} = X\ \underline{\beta} + \underline{\varepsilon}$ Where Y is the $(n \times 1)$ response vector and X is the $n \times (k + 1)$ design matrix, β is the $(k + 1) \times 1$ parameters vector (regression coefficients),

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and ε is the $(n \times 1)$ vector and $(Y \mid X, \beta, \sigma^2) \sim N(X\beta, \sigma^2 I_n)$. The normal conjugate prior density function of the parameters(β) defining the normal distribution is known in this situation as follows (Martz, H. F. & Krutchkoff, R.G. (1969))& (Harrison et al., 1989):

First case if (σ^2) is known

$$\beta \sim N(\beta_0, \sigma^2 M_0^{-1})$$

The kernel of the prior probability density function is:

$$f(\beta) \propto \exp\left[\frac{-1}{2\sigma^2}(\beta-\beta_0)' M_0 (\beta-\beta_0)\right] \quad -\infty < \beta < \infty, \ \sigma^2 > 0 \qquad \ldots(4)$$

Where:

$M_0 = X_0' X_0$: Informative Prior matrix.

β_0: mean of the Prior distribution.

$\sigma^2 M_0^{-1}$: Variance and covariance matrix of the Prior distribution.

As for the likelihood function, it is (Raftery, E., Maigan, D. & Hoeting, J.A. (1997)):

$$L(\beta,\sigma^2) \propto \exp\left[\frac{-1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\right] \qquad -\infty < Y < \infty \quad , \quad \sigma^2 > 0$$

$$\begin{aligned}(Y - X\beta)'(Y - X\beta) &= Y'Y - \beta' X'Y - Y'X\beta + \beta' X'X\beta \\ &= Y'X(X'X)^{-1}(X'X)(X'X)^{-1}X'Y - \beta'(X'X)(X'X)^{-1}X'Y - Y'X(X'X)^{-1}(X'X)\beta + \beta'X'X\beta \\ &= \hat{\beta}'_{OLS}(X'X)\hat{\beta}_{OLS} - \beta'(X'X)\hat{\beta}_{OLS} - \hat{\beta}'_{OLS}(X'X)\beta + \beta'X'X\beta \\ &= (\beta - \hat{\beta}_{OLS})'(X'X)(\beta - \hat{\beta}_{OLS})\end{aligned}$$

$$L(\beta,\sigma^2) \propto \exp\left[\frac{-1}{2\sigma^2}(\beta - \hat{\beta}_{OLS})'(X'X)(\beta - \hat{\beta}_{OLS})\right] \qquad \ldots(5)$$

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The function (4) can be combined with the function (5) to generate the posterior probability density function of the parameter vector (β) using Bayes' theorem as follows (Zellner, A. (1971):

$$f(\beta|Y,\sigma) \propto f(\beta)\,L(\beta,\sigma^2)$$

$$\propto \exp\left[\frac{-1}{2\sigma^2}(\beta-\hat{\beta}_{OLS})'(X'X)(\beta-\hat{\beta}_{OLS})+(\beta-\beta_0)'M_0(\beta-\beta_0)\right]$$

$$\propto \exp\left[\frac{-1}{2\sigma^2}(\beta-\hat{\beta}_B)'(X'X+M_0)(\beta-\hat{\beta}_B)\right], \quad -\infty<\beta<\infty, \qquad ....(6)$$

$$\hat{\beta}_{Bayse} = (X'X+M_0)^{-1}(X'Y+M_0\beta_0) \qquad ...(7)$$

Second case if (σ^2) is unknown

$$f(\beta,\sigma^2)=f(\beta|\sigma^2)f(\sigma^2)=N(\beta_0,\sigma^2M_0^{-1})*IG(\frac{v_0}{2},\frac{v_0S_0^2}{2})=NIG(\beta_0,\sigma^2M_0^{-1},\frac{v_0}{2},\frac{v_0S_0^2}{2})$$

According to the probability function below, the vector of the prior parameters is distributed in a multivariate-normal distribution (Shelemyahu Z., (1971)):

$$f(\beta|\sigma^2)=\frac{1}{(2\pi)^{\frac{p_0}{2}}\left|M_0^{-1}\right|^{\frac{1}{2}}}(\sigma^2)^{-\frac{p_0}{2}}\exp\left[\frac{-1}{2\sigma^2}(\beta-\beta_0)'M_0(\beta-\beta_0)\right], \quad \sigma^2>0, \quad -\infty<\beta<\infty \qquad .....(8)$$

Where:

$$p_0=k_0+1$$

$$\beta_0=(X_0'X_0)^{-1}X_0'Y_0$$

$$f(\sigma^2)=\frac{(\frac{v_0S_0^2}{2})^{\frac{v_0}{2}}}{\Gamma(\frac{v_0}{2})}(\sigma^2)^{-(\frac{v_0}{2}+1)}\exp\left[\frac{-v_0S_0^2}{2\sigma^2}\right], \quad \sigma^2>0, \quad v_0>0 \qquad .....(9)$$

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Additionally, it is expected that the probability function given below describes the (σ^2) distributed inverse gamma distribution:

Where:

$$v_0 S_0^2 = Y_0'Y_0 - \beta_0' X_0' Y_0$$

$$v_0 = n_0 - k_0 - 1 = n_0 - p_0$$

The following formula is used to construct a prior probability density function for the parameters (β, σ^2):

$$f(\beta, \sigma^2) \propto f(\sigma^2) f(\beta | \sigma^2)$$

$$f(\beta, \sigma^2) \propto (\sigma^2)^{-(\frac{v_0 + p_0}{2} + 1)} \exp\left[\frac{-1}{2\sigma^2}[v_0 S_0^2 + (\beta - \beta_0)' M_0 (\beta - \beta_0)]\right] \quad ....(10)$$

As for the likelihood function, it is:

$$L(\beta, \sigma^2) \propto (\sigma^2)^{-\frac{n}{2}} \exp\left[\frac{-1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\right] \qquad -\infty < Y < \infty \quad , \quad \sigma^2 > 0$$

We determine the posterior probability density function for the parameters using Bayes' theorem, and it looks like this (O'Hagan, A. (1973)):

$$f(\beta, \sigma^2 | Y) \propto f(\beta, \sigma^2) L(\beta, \sigma^2)$$

$$f(\beta, \sigma^2 | Y) \propto (\sigma^2)^{-(\frac{n + p_0 + v_0}{2} + 1)} \exp\left[\frac{-1}{2\sigma^2}[v_0 S_0^2 + (\beta - \beta_0)' M_0 (\beta - \beta_0) + (Y - X\beta)'(Y - X\beta)]\right]$$

$$f(\beta, \sigma^2 | Y) \propto (\sigma^2)^{-(\frac{n + p_0 + v_0}{2} + 1)} \exp\left[\frac{-1}{2\sigma^2}[v_0 S_0^2 + v S_e^2 + (\beta - \beta_0)' M_0 (\beta - \beta_0) + (\beta - \hat{\beta}_{OLS})'(X'X)(\beta - \hat{\beta}_{OLS})]\right]$$

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$$\propto (\sigma^2)^{-(\frac{f+p_0}{2}+1)} \exp\left[\frac{-1}{2\sigma^2}[f\,S_B^2 + (\beta-\hat{\beta}_B)'(M_0 + X'X)(\beta-\hat{\beta}_B)]\right] \qquad \text{..... (11)}$$

Where:

$$f = n + v_0$$

$$f\,S_B^2 = v_0 S_0^2 + v S_e^2$$

Therefore, the kernel of a normal-inverse gamma is represented by the formula (11). The formula (11) can be explained as follows:

$$(\beta,\sigma^2|Y) \sim MV\ NIG(\hat{\beta}_B, (M_0 + X'X)^{-1}, \frac{f}{2}, \frac{f\,S_B^2}{2})$$

So, according to (Gero W. and Thomas A. (2009)), the posterior probability density function is as follows:

$$f(\beta,\sigma^2|Y) = \frac{(\frac{f\,S_B^2}{2})^{\frac{f}{2}}}{(2\pi)^{\frac{p_0}{2}} \left|(M_0 + X'X)^{-1}\right|^{\frac{1}{2}} \Gamma(\frac{f}{2})} (\sigma^2)^{-(\frac{f+p_0}{2}+1)} \exp\left[\frac{-1}{2\sigma^2}\left[f\,S_B^2 + (\beta-\hat{\beta}_B)'(M_0 + X'X)(\beta-\hat{\beta}_B)\right]\right] \qquad \text{...(12)}$$

The posterior probability function of the parameter vector is obtained by integrating the function (12) with respect to the parameter (σ^2) as follows (Box, G.E.P, & Tiao, G. ,(1992)):

$$f(\beta|Y) = \int_0^\infty p(\beta,\sigma^2|Y)\, d\sigma^2$$

$$= \int_0^\infty MV\ NIG(\hat{\beta}_B, (M_0 + X'X)^{-1}, \frac{f}{2}, \frac{f\,S_B^2}{2})\, d\sigma^2 = M\ V\ St(\hat{\beta}_B,\ S_B^2 (M_0 + X'X)^{-1}, f)$$

$$f(\beta|Y) = \frac{\Gamma(\frac{f+p_0}{2})}{\Gamma(\frac{f}{2})(\pi)^{\frac{p_0}{2}} \left|f\,S_B^2 (M_0 + X'X)^{-1}\right|^{\frac{1}{2}}} \left[1 + \frac{(\beta-\hat{\beta}_B)'(M_0 + X'X)(\beta-\hat{\beta}_B)}{f\,S_B^2}\right]^{-[\frac{f+p_0}{2}]} \qquad \text{.....(13)}$$

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The (Multivariate -t-Distribution) formula (13) stands for this.

$$f(\sigma^2|Y) = \frac{f(\beta,\sigma^2|Y)}{f(\beta|Y,\sigma^2)}$$

$$= \frac{MVN \quad IG[(\hat{\beta}_B),(M_0 + X'X)^{-1}, \frac{f}{2}, \frac{f S_B^2}{2}]}{MVN \quad [(\hat{\beta}_B), f S_B^2 (M_0 + X'X)^{-1}]} \sim IG[\frac{f}{2}, \frac{f S_B^2}{2}]$$

$$f(\sigma^2|Y) = \frac{[\frac{f S_B^2}{2}]^{\frac{f}{2}}}{\Gamma[\frac{f}{2}]}[\sigma^2]^{-(\frac{f}{2}+1)}.\exp\left[\frac{f S_B^2}{2\sigma^2}\right] \qquad 0 < \sigma^2 < \infty \qquad .....(14)$$

For the parameter (σ^2) that represents the mode of the posterior probability density function (inverse gamma distribution), a Bayesian estimator may be created to (σ^2):

$$\hat{\sigma}_B^2 = \frac{f}{f+2} S_B^2 \qquad .....(15)$$

6: Application-side (simulation, real data) and outcome

To compare the efficiency of the Bayesian based on posterior mode method with the classical method of ordinary least squares in estimating a multiple linear regression model, estimation methods were applied first using the simulation method to simulate the greatest number of situations that can be encountered in practice in order to achieve more general results, and then using real data.

6.1: Description and analysis of simulation experiment

Experiment simulation is used to take the combination under consideration and repeat it (1000) times for different instances, as seen below:

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1- There are three sample sizes used: 50, 100, and 200.
2- Standard deviation, which can be (1, and 5).
3- The explanatory variables independently from a normal distribution, where k is number of the explanatory variables, which can be (2, 5, and 10).

Table 1: results of Simulation experiments for Bayesian posterior mode and OLS

		Sample size(**n**)	OLS	Bayesian posterior mode
			RMSE	**RMSE**
σ = 1	**k**=2	50	8.0466	3.2256
		100	8.7734	3.8777
		200	8.4091	2.6913
	k=5	50	8.9601	5.2086
		100	9.1328	4.1984
		200	8.9786	4.2774
	k=10	50	8.9467	5.1212
		100	9.1216	4.1335
		200	8.9783	4.2228
σ = 5	**k**=2	50	9.3886	4.6845
		100	9.9600	4.5548
		200	9.7276	4.3763
	k=5	50	10.1968	6.2432
		100	10.3552	5.4227
		200	10.2237	5.4918

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	k=10	50	9.8444	5.5316
		100	10.3437	5.3765
		200	10.1968	6.2432

From Tables (1) we noted all cases the Bayesian posterior mode approach of the value (RMSE) is less than the classic approach (OLS).

6.2: Descriptive and analysis of real data

Therefore, to apply the Bayesian approach and classical regression methods, data related to studies of ammonia-to-nitric acid oxidation. The data comes from a study by Rousseeuw and Leroy (1987). It was used to estimate multiple linear regression models.

The variables used in the study were described as follows:

First, the response variable:

Yi: stackloss.

Second: Explanatory variables (independent):

X1: The pace.

X2: temperature.

X3: acid concentration.

Table 2: presents statistical descriptions for all variables.

variables	y	x1	x2	x3
mean	17.65	57.90	21.15	86.05
Std. Deviation	10.419	13.262	3.233	5.385

Next, perform a multiple regression analysis using either the frequentist or OLS methods to see if all independent variables have an impact on the dependent variable.

Table 3: Multiple Regression Model Fitting (O LS) approach

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics Tolerance	
	B	Std. Error	Beta				
(Constant)	-51.605	19.639		-2.628	.018		
x1	.117	.148	.149	.788	.442	.390	2.565
x2	2.343	.644	.727	3.639	.002	.347	2.881
x3	.150	.252	.078	.597	.559	.818	1.222

When we contrast these variables' p-values with a level of significance ($\alpha = 0.01$), Demonstrating that these variables' p-values (x2) are smaller than (0.01) leads to the conclusion that they had an impact on the quantity of (y) values in Table 3. This indicates that all the estimated models do not suffer from the problem of multicollinearity between the explanatory variables because the values of (VIF) are less than (10).

Table 4: Model Summary

R Square	Adjusted R Square	Std. Error Estimate	Durbin-Watson
.778	.737	5.346	2.869

Tble (4) R-squared indicates that the fitted model explains 77% of the variation in (y). Since the corrected R square is 73%, it is more appropriate for comparing models with different numbers of independent variables. Because the value of (D.W.) calculated for the estimated models is within the range of accepting the null hypothesis, which is mentioned, the results shown in Table 4 show that none of the estimated functions have enough evidence to support the conclusion that there is a problem with autocorrelation between the values of the residuals. There isn't any first-order autocorrelation in the residual values.

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Coldfield-Quandt test

According to this test, we do not have sufficient evidence to conclude that heteroscedasticity is present in the regression model. Based on the F-Cal value, it is equal to (1.0181), which is less than the F-tab value, which is equal to (1.65).

Table 5: ANOVA table

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	1605.274	3	535.091	18.723	001
Residual	457.276	16	28.580		
Total	2062.550	19			

Table (5) displays the analysis of variance for the ANOVA table. The outcome shows that the sum squares for the regression, residual, and total are, respectively, (1605.274, 457.276, and 2062.550). Additionally, the F is 18.723, and the mean square for the regression and residual is 535.091, 28.580.

Table 6: Tests of Normality

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
y	.211	20	.020	.832	20	.033
x1	.229	20	.007	.837	20	.041
x2	.197	20	.041	.891	20	.028
x3	.220	20	.012	.910	20	.062

The understanding of the data's normalcy is one of the basic assumptions prior to analysis. This in Table 1 can be enhanced.

Table 7: results of real data for Bayesian posterior mode and OLS

Approaches		RMSE	R-Square
Classic Approach	OLS	5.347	0.77
new Approach	Bayesian posterior mode	2.185	0.92

Table 7 shows the following information:

The technique (Bayesian posterior mode) has a smaller root mean square error (RMSE) than the traditional approach.

7: conclusions

The study looked at the data, interpreted it, and came to a variety of conclusions. The most important ones are highlighted in the list of conclusions below:

1- For estimating parameters in the multiple linear regression model according to the statistical criterion, RMSE, the Bayesian technique based on posterior mode produced the best results.

2- A positive indication was obtained by the Bayesian method based on posterior mode in real data: a high coefficient of determination R-square.

3- The posterior mode-based Bayesian technique can also be used to estimate parameters for multiple linear regression models. After being used in all scenarios, a simulation study yielded favorable results.

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پەرەپێدانی تەکنیکێکی بایيزی بە بەکارهێنانی پشتەوە لەسەر بنەمای شێواز بۆ خەملاندنی پارامێتەرەکان لە پاشەکشەی هێڵی فرەییدا (توێژینەوەی هاوشێوەکردن)

پـوختـە:

لەم لێکۆڵینەوەیەدا، ئێمە تەکنیکێکی بەیزمان پەرەپێدا کە شێوازێکی بنەمادار بە دواوە بەکاردەهێنێت بۆ خەملاندنی پارامێتەرەکان لە پاشەکشەی هێڵی فرەییدا. باشترین مۆدێلی پاشەکشەی هێڵی فرەیی بۆ داتاکان ڕەنگە بە پشتبەستن بە هەڵەی چوارگۆشەی مامناوەند بەدەست بهێنرێت دوای بەراوردکردنی پشتەوەی بایيزی لەسەر بنەمای شێواز و شێوازی تەقلیدی (کەمترین چوارگۆشەی ئاسایی) بە تێکەڵکردنی داتا هاوشێوەکراوەکان و ڕاستەقینەکان لەگەڵ بەرنامەیەکی MATLAB کە بە تایبەتی بۆ ئەم مەبەستە دروستکراوە . توێژینەوەکە دەریدەخات کە، بە بەراورد لەگەڵ ڕێبازی تەقلیدی، پشتەوەی بەیزی لەسەر بنەمای ڕێبازی شێواز خەملاندنی وردتری پارامێتەرەکان بەدەست دەهێنێت.

تطوير تقنية بيزي باستخدام أسلوب لاحق بناءً على الوضع لتقدير المعلمات في الانحدار الخطي المتعدد (دراسة المحاكاة)

الملخص:

في هذه الدراسة ، قمنا بتطوير تقنية بيز باستخدام الوضع الخلفي لتقدير المعلمات في الانحدار الخطي المتعدد. يمكن الحصول على أفضل نموذج انحدار خطي متعدد للبيانات بناءً على جذرتربيعي لمتوسط خطأ بعد مقارنة الخلفية البيزية بناءً على الوضع والطريقة التقليدية (المربعات الصغرى الاعتيادية) من خلال الجمع بين البيانات المحاكاة والحقيقية مع برنامج MATLAB المصمم خصيصًا لهذا الغرض . توصلت الدراسة إلى أنه مقارنة بالنهج التقليدي ، فإن الخلفية البيزية بناءً على نهج الوضع ينتج عنها تقديرات معلمات أكثر دقة.