

## Modeling and Forecasting the Volatility of Gasoline Prices Using Symmetric and Asymmetric GARCH Models in Erbil City

**Dr. Luceen Immanuel Kework**

Statistics Department/ College Administration and Economic / Salahaddin University –Erbil

[Luceen.kework@su.edu.krd](mailto:Luceen.kework@su.edu.krd)

**Lecturer Dona diya butros**

Statistics Department/ College Administration and Economic / Salahaddin University –Erbil

[donastat90@gmail.com](mailto:donastat90@gmail.com)

### ARTICLE INFO

#### **Article History:**

Received: 15/11/2018

Accepted: 15/1/2019

Published: Spring 2019

Doi:

**10.25212/lfu.qzj.4.2.18**

#### **Keywords:**

*Conditional Variance; Volatility Clustering; Symmetric and Asymmetric GARCH Models; Error Distribution; Volatility Forecasting; Root Mean Square Error (RMSE)..*

### ABSTRACT

The paper aims to compare the performance of several univariate symmetric and asymmetric GARCH volatility models in modeling and forecasting the volatility of daily Gasoline prices in Erbil city. This paper chooses the GARCH, GARCH-M, TGARCH, E-GARCH and Power GARCH model to analyze the daily return of Gasoline under three different error distributions: normal distribution, student-t distribution and generalized error distribution and then compare the results and choose the appropriate model to forecast the volatility. The sample is divided into two subsamples. The first subsample is called in-sample data set (Training sample) used to estimate the ARMA-GARCH models for underlying data and the second subsample is called out-sample data set (Testing sample) used to investigate the performance of volatility forecasting. As a result of analyses, we conclude that the best model fits the

volatility of Gasoline returns series is AR(2)-Power GARCH(2,1,1) non-linear asymmetric model with innovation student-t distribution (d.f =10), and has better forecasting performance than others models. This result is important in many fields of finance such as investment decisions, asset pricing, portfolio allocation and risk management.

**Keywords:** Conditional Variance; Volatility Clustering; Symmetric and Asymmetric GARCH Models; Error Distribution; Volatility Forecasting; Root Mean Square Error (RMSE)..

## **INTRODUCTION**

Volatility refers to the amount of uncertainty or risk about the size of changes in a security's value. A higher volatility means a security's value can potentially be spread out over a larger range of values whereas, lower volatility means a securities value does not fluctuate dramatically, but changes in value over a period of time. Volatility is defined as the fluctuations in assets prices. As a barometer of the market risk, volatility is important for investment decisions, asset pricing, portfolio allocation and risk management in finance. In this respect, it is crucial to forecast volatility accurately in finance literature. Over the last few years, modeling volatility of a financial time series has become an important area and has gained a great deal of attention from academics, researchers and others. The time series are found to depend on their own past value (autoregressive), depending on past information (conditional) and exhibit non-constant variance (Heteroscedasticity). It has been found that the market volatility changes with time (i.e., it is 'time-varying') and exhibits 'volatility clustering.' A series with some periods of low volatility and some periods of high volatility is said to exhibit volatility clustering. Associated with the increasing importance of volatility, different volatility models come into use in the finance literature. Conditional heteroscedasticity models are the most commonly used volatility models in forecasting financial assets volatility. In volatility forecasting

(Engle 1982) Autoregressive Conditional Heteroscedasticity Model (henceforth ARCH) and (Bollerslev 1986) Generalized Autoregressive Heteroscedasticity Model (henceforth GARCH) is being used in the literature.

The best idea is to estimate ARMA-GARCH models in-sample periods and selection the best volatility models for the daily Gasoline prices data, depending on less value of (Akaike information criterion and Schwartz information criterion), also the parameters must be significant, in addition the residuals don't have the serial correlation and ARCH effect, as well as these models should have the higher value of log-likelihood. The effect of the random error type of models was also examined, by studying three types of statistical distributions (Normal, GED and Student-t). Finally, we evaluated out-of-sample forecasting performance of the volatility models, and then choose the best model to forecast the volatility of daily Gasoline prices returns data.

The paper aims to compare the performance of univariate symmetric and asymmetric GARCH models in modeling and forecasting the volatility of daily Gasoline prices in Erbil city. The volatility models applied in this paper include the AR(2)-GARCH(1,1), AR(2)-GARCH-M(1,1), AR(2)-TGARCH(1,1,1), AR(2)-EGARCH(1,1,1) and AR(2)-Power GARCH(2,1,1). Future market is important in terms of reducing the uncertainty about the future, forecasting the future values of prices, providing efficient risk management. This paper focus on forecasting volatility in future market. Therefore, the findings of paper will contribute to the existing literature.

The paper is organized as follows: In the section 1, it's a brief introduction. Section 2 describes the theoretical side of methodology used. The data analysis and explains the dataset used and the out-of-sample forecast is presented in Section 3, section 4 points out the conclusion. The reference and the appendixes can be found at the end.

## **2. Theoretical Side**

### **2.1 Time series model**

A time series is a set of observations on a variable representing one entity over  $t$  periods of time (Kirchgässner, Wolters et al. 2012). There are two types of time series, linear and non-linear. Firstly we discuss some simple time series models, that are useful in modeling the mean equation and then we

introduce some nonlinear models that are applicable to financial time series (Tsay 2002).

**2.1.1 Linear models:**

In linear models, we firstly use 1- simple autoregressive (AR) models, 2- simple moving-average (MA) models, 3- mixed autoregressive moving-average (ARMA) models. To identify the best fitted model in modeling unconditional mean equation.

**2.1.1.1 Autoregressive Model (AR)**

Autoregressive processes are as their name suggests regressions on themselves. Specifically, a p-th order autoregressive process  $y_t$  satisfies the equation:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + u_t \text{ or } y_t = \sum_{i=1}^p \varphi_i y_{t-i} + u_t \quad \dots, (1.1)$$

Where,  $y_t$  is a linear combination of the p most recent past values of itself plus an “innovation” term  $u_t$  that incorporates everything new in the series at time t (Cryer and Chan 2008).

**2.1.1.2. Moving Average (MA) Models**

We now turn to another class of simple models that are also useful in modeling return series in finance. These models are called moving-average of order q and abbreviate the name to (MA) models (Tsay 2002). The general form of an MA (q) model is (Cryer and Chan 2008):

$$y_t = u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \dots - \theta_q u_{t-q} \text{ or } y_t = u_t - \sum_{i=1}^q \theta_i u_{t-i} \quad \dots, (1.2)$$

**2.1.1.3. Autoregressive Moving Average (ARMA) Processes**

By combining the AR (p) and MA (q) models, an ARMA (p, q) model is obtained (Brooks 2008). The model could be written as:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \dots - \theta_q u_{t-q} \dots, (1.3)$$

Or

$$y_t = \sum_{i=1}^p \varphi_i y_{t-i} + u_t - \sum_{i=1}^q \theta_i y_{t-i} \dots, (1.4)$$

We say that  $y_t$  is a mixed autoregressive moving average process of orders  $p$  and  $q$ , respectively; we abbreviate the name to ARMA ( $p, q$ ) (Cryer and Chan 2008).

### 2.1.2. Non-Linear Models (The ARCH Family Models: Volatility Modeling Techniques)

There are an infinite number of different types of non-linear model. The most popular financial models are the family of ARCH models used for modeling and forecasting volatility. It is unlikely in the context of financial time series that the variance of the errors will be constant over time. If the variance of the errors is not constant, this would be known as heteroscedasticity (Brooks 2008).

This study considers ARCH family models. The models of volatility can be divided into two main categories, symmetric(ARCH, GARCH and GARCH-M)the effect of errors on the conditional variance is symmetric, i.e., a positive error has the same effect as a negative error of the same magnitude, and asymmetric models (TARCH, EGARCH and PGARCH)the conditional variance depends on the sign(William and Shyong 1994).

#### 2.1.2.1 Symmetric Models

##### a. Autoregressive conditionally heteroscedastic (ARCH) model:

The first model that provides a systematic framework for volatility modeling is the ARCH model of (Engle 1982). They have been found useful in numerous applications, especially in the context of financial time series which often exhibit large variability. The formula of the ARCH ( $p$ ) model is:

$$y_t = \mu + \varepsilon_t \quad \text{mean equation} \quad \dots, (1.5)$$

$$\varepsilon_t = \sigma_t z_t \quad , \quad z_t \sim \text{iid}(0, 1) \quad \dots (1.6)$$

Where,  $y_t$  denote a stationary time series,  $\mu$  is the mean of  $y_t$ .  $\varepsilon_t$ : is independent and identically distributed (i.i.d.) with mean zero,  $\varepsilon_t \sim \text{iid}(0, \sigma_t^2)$ ,  $\sigma_t^2$  is the conditional variance of the innovations errors at time  $t$  and  $z_t$  is assumed to be i.i.d. standard normal in the basic ARCH model.

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \dots + \alpha_p \epsilon_{t-p}^2 \\ &= \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 \quad \dots, (1.7) \end{aligned}$$

Where,  $\alpha_0$  is the constant term  $\alpha_0 > 0$ ,  $\alpha_i$  is an ARCH term  $0 < \alpha_i < 1$ . Since  $\epsilon_t$  has a zero mean,  $\text{Var}_{t-1}(\epsilon) = E_{t-1}(\epsilon_t^2) = \sigma_t^2$ , the above equation can be rewritten as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \dots + \alpha_p \epsilon_{t-p}^2 + u_t \quad \dots, (1.8)$$

and the model in (1.5) and (1.7) is known as the autoregressive conditional heteroscedasticity (ARCH) model, which is usually referred to as the ARCH(p) model (Zivot and Wang 2006).

**b. Generalized Autoregressive conditionally heteroscedastic (GARCH) models**

The GARCH model was developed independently by (Bollerslev 1986). Who allows the conditional variance to be dependent upon previous own lags (Brooks 2008). The GARCH function takes two arguments: the first argument is the conditional mean equation, while the second argument is formula which specifies the conditional variance equation (Zivot and Wang 2006). The GARCH(p, q) model can be written as

$$\begin{aligned} y_t &= \mu + \epsilon_t \quad \text{mean equation} \quad \dots, (1.9) \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 \\ &+ \sum_{j=1}^q \beta_j h_{t-j} \quad \text{variance equation} \quad \dots, (1.10) \end{aligned}$$

Where, the coefficients  $\alpha_i$  ( $i = 0, \dots, p$ ) and  $\beta_j$  ( $j = 1, \dots, q$ ) are all assumed to be positive  $\alpha_0 \geq 0$ , the ARCH term  $\alpha_1 \geq 0$  and the GARCH term  $\beta_j \geq 0$  to ensure that the conditional variance  $\sigma_t^2$  is always positive.  $\alpha_1 u_{t-1}^2$  is information about volatility during the previous period,  $\beta_1 \sigma_{t-1}^2$  is the fitted variance from the model during the previous period. The general GARCH(p, q) model covariance stationarity requires  $\sigma_t = \alpha_0 + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-j} < 1$  (Zivot and Wang 2006)(Brooks 2008)(Gregoriou 2009).

**c. ARCH-in-Mean Model**

Engle, Lilien et al. 1987 extended the basic ARCH framework to allow the mean of a sequence to depend on its own conditional variance. This class of model, called the General Autoregressive Conditional Heteroscedasticity in Mean model (ARCH in mean or ARCH-M) model (Enders 2015) for estimating time-varying risk premiums with time-varying variances. The GARCH-M version of this model is more commonly used, and is specified as:

$$y_t = \mu + \delta\sigma_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{iid}(0, \sigma_t^2) \quad \dots, (1.11)$$

$$\sigma_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad \dots, (1.12)$$

Where the parameter  $\delta$  can be interpreted as the price of risk and can thus be assumed to be positive (Francq and Zakoian 2011).

**2.1.2.2 Asymmetric Models**

The asymmetric news impact is usually referred to as the leverage effect. It seems the bad news to have a more pronounced effect on volatility than good news. There is a strong negative correlation between the current return and the future volatility. This tendency for volatility to decline when returns rise and to rise when returns fall is often called the leverage effect(Enders 2015).

**a. The Exponential GARCH (EGARCH) Model**

Nelson 1991 proposed the following exponential GARCH (EGARCH) model to allow for leverage effects (Zivot and Wang 2006). The EGARCH (p, q) model specifies conditional variance in logarithmic form, which means that there is no need to impose an estimation constraint in order to avoid negative variance (Poon 2005):

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^q b_j \ln \sigma_{t-j}^2 \quad \dots, (1.13)$$

Where,  $\sigma_t^2$  is the conditional variance,  $\ln \sigma_t^2 = \log \sigma_t^2$ . Note that when  $\varepsilon_{t-i}$  is positive or there is good news, the total effect of  $\varepsilon_{t-i}$  is  $(1 + \gamma_i)|\varepsilon_{t-i}|$  in contrast, when  $\varepsilon_{t-i}$  is negative or there is bad news, the total effect of  $\varepsilon_{t-i}$  is  $(1 - \gamma_i)|\varepsilon_{t-i}|$  and the value of  $\gamma_i$  is asymmetric response parameter or leverage effect, would be expected to be negative (Zivot and Wang 2006).

**b. The Threshold GARCH (TGARCH) Model:**

Another GARCH variant that is capable of modeling leverage effects is the threshold GARCH (TGARCH) model. is also known as the GJR-GARCH model because (Glosten, Jagannathan et al. 1993) proposed essentially the same model, to allow for asymmetric effects of positive and negative shocks on volatility, which has the following form (Franses and Van Dijk 2000).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \gamma_i \varepsilon_{t-i}^2 S_{t-i} + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad \dots, (1.14)$$

Where;

$$S_{t-i} = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{if } \varepsilon_{t-i} \geq 0 \end{cases}$$

$\alpha_0 > 0$ ,  $\alpha_i > 0$  and  $\beta_j > 0$ . That is, depending on whether  $\varepsilon_{t-i}$  is above or below the threshold value of zero,  $\varepsilon_{t-i}$  has different effects on the conditional variance  $\sigma_t^2$ , when  $\varepsilon_{t-i}$  is positive, the total effects are given by  $\alpha_i \varepsilon_{t-i}^2$ , when  $\varepsilon_{t-i}$  is negative, the total effects are given by  $(\alpha_i + \gamma_i) \varepsilon_{t-i}^2$ . So one would expect  $\gamma_i$  to be positive for bad news to have larger impacts (Zivot and Wang 2006).

**c. The Power GARCH (PGARCH) Model:**

Ding, Granger et al. 1993 introduced the asymmetric power ARCH model also called PARCH to estimate the optimal power term if it satisfies an equation of the form (Francq and Zakoian 2011).

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta \quad \dots, (1.15)$$

Where,  $\alpha_0 > 0$ ,  $\alpha_i > 0$ ,  $\delta \geq 0$ ,  $\beta_j \geq 0$  and  $|\gamma_i| \leq 1$ ,  $\alpha_i$  is the ARCH term,  $\beta_j$  is the GARCH term,  $\delta$  is the parameter for the power term and  $\gamma_i$  are the leverage parameter. The power transformation is achieved by taking squaring operations of the residual or to the power of 2, it can possess richer volatility patterns such as asymmetry and leverage effects (Wang 2005)(Gregoriou 2009).

**2.2 The Distribution of Error**



The volatility changes randomly in time, has distributions with heavy or semi-heavy tails, and clusters on high levels. In this study we used different distributions for the error term like (normal distributions, Student-t distributions and generalized error distributions (GED)) (Gregoriou 2009).

**2.2.1 Normal distributions**

The normal distribution is very well known since it arises in many applications. The main importance of normal distribution lies on the central limit theorem which says that the sample mean has a normal distribution if the sample size is large. A random variable X is said to have a normal distribution if its probability density function is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \dots, -\infty < x > \infty \quad \dots, (1.16)$$

Where,  $\mu$  is the mean  $-1 < \mu > 1$  and  $\sigma^2$  is the variance  $0 < \sigma^2 > \infty$ . If x has a normal distribution with parameters  $\mu$  and  $\sigma^2$ , then we write  $X \sim N(\mu, \sigma^2)$ (Sahoo).

**2.2.2 Student’s t-distribution**

The Student’s t-distribution is one of the very useful sampling distributions. A continuous random variable x is said to have a t-distribution with v degrees of freedom if its probability density function is of the form:

$$f(x; v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{(\pi v)^{\frac{1}{2}} \Gamma\left(\frac{v}{2}\right) \left[1 + \frac{x^2}{v}\right]^{\frac{v+1}{2}}} \quad \dots, (1.17)$$

Where,  $-\infty < x > \infty$  and  $v > 0$ . If x has a t-distribution with v degrees of freedom, then we denote it by writing  $x \sim t(v)$  (Sahoo).

**2.2.3 Generalized Error Distribution**

Nelson 1991 proposed to use the generalized error distribution (GED) to capture the fat tails usually observed in the distribution of financial time series. If a random variable  $u_t$  has a GED with mean zero and unit variance, the PDF of  $u_t$  is given by:

$$f(u_t) = \frac{v \exp[-(\frac{1}{2})|\frac{u_t}{\lambda}|^v]}{\lambda \cdot 2^{\frac{v+1}{v}} \cdot \Gamma(\frac{1}{v})} \dots \text{ where, } \lambda$$

$$= \left[ \frac{2^{-\frac{2}{v}} \Gamma(\frac{1}{v})}{\Gamma(\frac{3}{v})} \right]^{\frac{1}{2}} \dots, (1.18)$$

Where,  $v$  is a positive parameter governing the thickness of the tail behavior of the distribution. When  $v = 2$  the above PDF reduces to the standard normal PDF, when  $v < 2$  the density has thicker tails than the normal density and when  $v > 2$  the density has thinner tails than the normal density (Zivot and Wang 2006).

### 2.3 Model Constructing Strategy

A simple way to construct an ARCH model consists of three steps: (1) construct an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence in the data, and use the residual series of the model to test for ARCH effects; (2) specify the ARCH order and perform estimation; and (3) check the fitted ARCH model carefully and refine it if necessary (Tsay 2002).

#### 2.3.1 Identification

The first step of model building is model identification. In this step we look at the time series plot, compute many different statistics from the data to know if the series is stationary or non-stationary. The model chosen at this point is tentative and subject to revision later on in the analysis. A non-stationary time series may exhibit a systematic change in mean, variance, or both. There are some intuitive ideas regarding dealing with non-stationary time series. For example, return series we take logs first and then compute first differences the order does matter. In financial literature, the differences of the (natural) logarithms are usually called returns (Cryer and Chan 2008).

$$\text{Return Series} = \log\left(\frac{y_t}{y_{t-1}}\right)$$

$$= \log(y_t) - \log(y_{t-1}) \dots, (1.19)$$

Where,  $d$  is the difference,  $\nabla Y_t = Y_t - Y_{t-1}$  is the first difference.

### 2.3.1.1 ARCH and GARCH Models Tests

Before estimating a full ARCH model of the mean equation for a financial time series, it is usually good practice to test for the presence of ARCH effects in the residuals. If there are no ARCH effects, then the ARCH model is unnecessary (Zivot and Wang 2006). An ARMA model is built for the observed time series to remove any serial correlations in the data. For most assets return series. For some daily return series, a simple AR, MA, or ARIMA model might be needed (Tsay 2002).

#### a. Unit Root Tests (Testing for Stationary)

To test whether these series have a unit root, it is important to take the kind of non-stationarity into account, i.e. to ask whether the series contains a deterministic or a stochastic trend when it comes to transforming non-stationary into stationary time series (Kirchgässner, Wolters et al. 2012) To test while the data is stationary, we performed the Augmented Dickey-Fuller (ADF) and Phillips-Perron tests (Brockwell, Davis et al. 2002).

#### 1) Augmented Dickey-Fuller (ADF) test

The augmented Dickey-Fuller (ADF) test statistic is the t-statistic of the estimated coefficient from the method of least squares regression. However, the ADF test statistic is not approximately t-distributed under the null hypothesis; instead, it has a certain nonstandard large-sample distribution under the null hypothesis of a unit root (Cryer and Chan 2008).

$H_0: y = 0$  (series is stationary) Vs  $H_1: y < 0$  (*series is not stationary*)

We apply the augmented Dickey-Fuller (ADF) test based on the OLS regression

$$\nabla y_t = \alpha_0 + \beta_t + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \nabla y_{t-i} + \varepsilon_t \quad \dots, (1.20)$$

Where,  $\nabla y_t = y_t - y_{t-1}$ ,  $\nabla$  means the difference of return series and  $(\alpha_0, \beta, \gamma, \delta)$  are the parameters. This test assumes that the residuals  $\varepsilon_t$  are independently and identically distributed (Gregoriou 2009). Since the absolute values of all t-statistics are well below this critical value, we cannot reject the null hypothesis of a unit root in any of the series at the 5% level (Enders 2015).

#### 2) Phillips-Perron (PP) tests

The Phillips-Perron (PP) unit root tests differ from the ADF tests mainly in how they deal with serial correlation and heteroscedasticity in the errors. In particular (Zivot and Wang 2006). The t-statistic of the PP test is calculated as:

$$t = \sqrt{\frac{r_0}{h_0}} t_0 - \frac{(h_0 - r_0)}{2h_0\sigma} \sigma_\theta \quad \dots, (1.21)$$

Where,  $h_0 = r_0 + 2 \sum_{j=1}^M (1 - \frac{j}{T}) r_j$ , Perron reports the following critical values of the t-statistic at the 5% significance level. Where,  $r_j$  is the autocorrelation function at lag  $j$ ,  $t_0$  is the t-statistic of  $\theta$ ,  $\sigma_\theta$  is the standard error of  $\theta$ , and  $\sigma$  is the standard error of the test regression. In fact,  $h_0$  is the variance of the  $m$ -period differenced series,  $y_t = y_{t-m}$ ; while  $r_0$  is the variance of the one-period difference,  $\nabla y_t = y_t - y_{t-1}$  (Wang 2005)(Enders 2015).

**b. Ljung-Box Test (Serial Correlation)**

There are several tests of randomness, the first test Ljung–Box statistics of the residuals can be used to check the adequacy of a fitted model. If the model is correctly specified, then  $Q_{(m)}$  follows asymptotically a chi-squared distribution with  $m-p$  degrees of freedom, where  $p$  denotes the number of parameters used in the model. The test statistic is:

$$Q_{(m)} = n(n + 2) \sum_{k=1}^m \frac{\hat{p}_k^2}{n - k} \sim \chi_{m-p}^2 \quad \dots, (1.22)$$

Where,  $\hat{p}_k$  is the lag  $k$  autocorrelation of the absolute standardized residuals,  $n$  is the sample size and  $m$  number of lags of autocorrelation. Notice that since  $(n + 2)/(n - k) > 1$  for every  $k \geq 1, n \rightarrow \infty$ . We would reject the null hypothesis at level  $\alpha$  if the value of  $Q$  exceeds ( $p$ -value  $< 0.05$ ). The hypothesis is written as (Cryer and Chan 2008) (Tsay 2002) (Shumway and Stoffer 2000):

$H_0$ : there is no serial correlation      Vs       $H_1$ : there is serial correlation

**c. Lagrange Multiplier Test (ARCH effect) Testing for Heteroscedasticity**

The second one is the Lagrange Multiplier test. Before estimating a full ARCH model for a financial time series, it is usually good practice to test for the presence of ARCH effects in the residuals (Zivot and Wang 2006). The corresponding LM test can be computed as:

$$LM = nR^2 \sim \chi^2_p \quad \dots, (1.23)$$

The LM test-statistic has an asymptotic  $\chi^2_p$  distribution. Where  $n$  is the sample size and the  $R^2$  is obtained from a regression of the squared residuals on a constant and  $p$  of its lags,

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \dots + \alpha_p \hat{\epsilon}_{t-p}^2 + e_t \quad , t = p + 1, \dots, T \quad \dots, (1.24)$$

Where, the residuals  $\hat{\epsilon}_t$  are obtained by estimating the model for the conditional mean of the observed time series  $y_t$  and  $T$  is the sample size. In this case, the  $p$ -value is essentially zero, which is smaller than the conventional 5% level, so reject the null hypothesis that there are no ARCH effects under the null hypothesis (Franses and Van Dijk 2000)(Tsay 2002)(Zivot and Wang 2006).

$H_0$ : there is no ARCH effects Vs  $H_1$ : there is ARCH effects

**d. Leverage effect**

The GARCH model is characterized by asymmetric response of current volatility to positive and negative lagged errors  $u_{t-1}$  (Lütkepohl, Krätzig et al. 2004). It could be interpreted fittingly as a measure of news entering a financial market in time  $t$ . This tendency for volatility to decline when returns rise and to rise when returns fall is often called the leverage effect. However, one way to test for leverage effect (asymmetric effect) is to estimate the TAR, EGARCH or PGARCH model (Zivot and Wang 2006)(Enders 2015).

**e. Jarque-Bera (J-B) Statistic, Test for Normality**

The Jarque-Bera test is tests the residuals of the fit for normality based on the result that a normally distributed random variable has skewness equal to zero and kurtosis equal to three. The Jarque-Bera test statistic is (Zivot and Wang 2006):

$$JB = \frac{n}{6} \left( \widehat{skew}^2 + \frac{(\widehat{kurt} - 3)}{4} \right) \quad , JB \sim \chi^2 \quad \dots (1.25)$$

We reject the hypothesis of normally distributed errors if a calculated value of the statistic exceeds a critical value selected from the chi-squared distribution with two degrees of freedom

$H_0$ : The residual series are normal distribution

$H_1$ : The residual series are non – normal distribution

### **2.3.2 Estimation Methods of Parameters:**

In this case, when you fit a linear regression on time series data, the parameters in the model for the conditional mean can be estimated in a first step by least squares. In a second step, the parameters in the GARCH model are estimated with maximum likelihood for the variance equation, using the residuals  $\hat{\epsilon}_t$  obtained in the first step (Franses and Van Dijk 2000).

#### **Maximum Likelihood Estimation**

After diagnostics the model of time series data, the parameters of ARCH family models are estimated by the maximum likelihood method. The function can be written as:

$$L = -\frac{N}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^N \log \sigma_t^2 - \frac{1}{2} \sum_{t=1}^N \frac{\epsilon_t^2}{\sigma_t^2} \quad \dots, (1.26)$$

Once the MLE estimates of the parameters are found, estimates of the time varying volatility  $\sigma_t$  ( $t = 1, \dots, T$ ) are also obtained (Satchell and Knight 2011)(Zivot and Wang 2006).

### **2.3.3 Model Checking**

Before we accept a fitted model, it is necessary to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data. If some basic model assumptions seem to be violated, then a new model should be specified; fitted, and checked again until a model is found that provides an adequate fit to the data (Cryer and Chan 2008).

#### **1) Significance of model parameters**

All parameter estimates by least squares and maximum likelihood must be highly significant with p-values (Brooks 2008).

#### **2) Checks of the Standardized Residual (Serial Correlation)**

The squares of the standardized residuals were checked for serial correlation. The estimated residuals should be serially uncorrelated and should not display any remaining conditional volatility. If there is no serial correlation in the residuals, the autocorrelations and partial autocorrelations at all lags should be nearly zero, and all Q-statistics should be insignificant with large p-values. To

test the model of the mean, form the Ljung–Box Q-statistics for the sequence up to a specific lag. You should not be able to reject the null hypothesis(Wang 2005).

$H_0$ : there is no serial correlation

Vs  $H_1$ : there is serial correlation

Simply divide  $\hat{\epsilon}_t$  by  $\hat{h}_t$  in order to obtain an estimate of what we have been calling the  $v_t$  sequence. Since  $\epsilon_t$  have a zero mean and a variance of  $h_t$ , you can think of  $v_t = \epsilon_t/(h_t)^{1/2}$  as the standardized value of  $\epsilon_t$ (Enders 2015).

### **3) Lagrange Multiplier Test (ARCH effect)**

A test for determining whether ARCH effects are remaining in the residuals of an estimated model may be conducted. The test can also be thought of as a test for autocorrelation in the squared residuals. If the value of the test statistic is less than the critical value from the  $\chi^2$  distribution, then accept the null hypothesis that the sample values of the Q-statistics are equal to zero (Brooks 2008).

$H_0$ : there is no ARCH effects Vs  $H_1$ : there is ARCH effects

Form the Ljung–Box Q statistics of the squared standardized residuals (i.e.,  $s_t^2$ ). The basic idea is that  $s_t^2$  is an estimate of  $v_t^2 = \epsilon_t^2/h_t$ . Hence, the properties of the  $s_t^2$  sequence should mimic those of  $v_t^2$ , the properties of the  $s_t^2$  sequence should mimic those of  $v_t^2$  (Enders 2015).

### **4) Model selection criteria**

Most of the methods used in the literature for model selection are based on evaluating the ability of the models to describe the data. An important practical problem is the determination of the ARCH order  $p$  and the GARCH order  $q$  for a particular series. Since GARCH models can be treated as ARMA models for squared residuals, On the other hand, the most frequently used in-sample methods of model evaluation are the information criteria. Standard model selection criteria such as the Akaike information criterion and the Schwartz Information Criterion can be used for selecting models that best fitting the data (Xekalaki and Degiannakis 2010)(Andersen, Davis et al. 2009).

#### **1) Akaike Information Criterion**

Akaike information criterion (AIC) has been used in the literature on ARCH models for selecting the appropriate model specification. The model

corresponding to the minimum value of the criterion is referred to be the best-performing one. These criteria are defined as follows:

$$AIC = n \ln \hat{\sigma}^2 + 2h \quad \dots, (1.27)$$

Where,  $\hat{\sigma}^2$  is the estimator of the variance,  $h$  is the number of parameters in the model and  $n$  is the sample size (Brooks 2008)(Xekalaki and Degiannakis 2010).

## **2) Schwartz Information Criterion**

The same rule applies to the Schwarz criterion, for determining the appropriate model should be chosen the lowest value of SIC. We use the following formulas:

$$SIC = n \ln \hat{\sigma}^2 + h \ln(n) \quad \dots, (1.28)$$

Where,  $\hat{\sigma}^2$  is the estimator of the variance,  $h$  is the number of parameters in the model and  $n$  is the sample size. The SIC penalizes additional parameters more heavily than the AIC because  $\ln n > 2$  for  $n > 8$ . Therefore, the model order selected by the SIC is likely to be smaller than that selected by the AIC (Brooks 2008)(Franses and Van Dijk 2000).

### **2.3.4 Forecasting (In-Sample and Out-of-Sample)**

Forecasting is an important application of time series analysis, the goal is to predict future volatility of a time series, based on the data collected to the present. In this context, the decisions made today will reflect forecasts of the future state of the world. In all forecast evaluations, it is important to distinguish in-sample and out-of-sample forecasts. In-sample forecast, which is based on parameters estimated using all data in the sample, implicitly assumes parameter estimates are stable across time. One would expect the 'forecasts' of a model to be relatively good in-sample, for this reason. Therefore, a sensible approach to model evaluation through an examination of forecast accuracy is not to use all of the observations in estimating the model parameters, but rather to hold some observations back. The latter sample, sometimes known as a holdout sample, would be used to construct out-of-sample forecasts. A good forecasting model should be one that can withstand the robustness of out-of-sample test, that is closer to reality It is customary to evaluate forecasting model performance using the one-step-ahead forecast errors (Brooks 2008)(Poon 2005).



### **2.3.4.1 Evaluation of volatility forecasting performance**

Comparing forecasting performance of competing models is one of the most important aspects of forecasting exercise. We consider how to evaluate the performance of a forecasting technique for a particular time series. Concerning the forecast errors, there are four useful statistical measures that describe how well the model fits. These forecast accuracy measures can also be used to discriminate between competing models (Brooks 2008)(Montgomery, Jennings et al. 2015)(Poon 2005).

#### **1) Root Mean Square Error (RMSE)**

Every forecast error gets the same weight in this measure. The root mean square error is often used to give particularly large errors a stronger weight (Kirchgässner, Wolters et al. 2012).

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (\hat{\sigma}_t - \sigma_t)^2} \quad \dots, (1.29)$$

Where,  $\hat{\sigma}_t^2$  is one step ahead volatility forecast,  $\sigma_t^2$  is the actual volatility and N is the number of forecasts (Poon 2005).

#### **2) Mean Absolute Error (MAE)**

Mean absolute error (MAE) measures the average absolute forecast error when ignoring signs. (Brooks 2008)(Armstrong 2001)(Poon 2005) , and is given by

$$MAE = \frac{1}{N} \sum_{t=1}^N |\hat{\sigma}_t - \sigma_t| \quad \dots, (1.30)$$

#### **3) Mean Absolute Percent Error (MAPE)**

Mean absolute percentage error is the average absolute percentage change between the predicted value for a one-step-ahead forecast and the true value, taken without regard to sign (Armstrong 2001)(Montgomery, Jennings et al. 2015)(Poon 2005), is given as

$$MAPE = \frac{1}{N} \sum_{t=1}^N \frac{|\hat{\sigma}_t - \sigma_t|}{\sigma_t} \quad \dots, (1.31)$$

#### **4) Thiel's U-test**

The Theil inequality coefficient is the scaled measure that always lies between zero and one. If the forecasts are good then U should be less than one.(Poon 2005)(Armstrong 2001).

$$\text{Thiel's } U = \left[ \sum_{t=1}^N (\hat{\sigma}_t - \sigma_t)^2 \right]^{1/2} / \left[ \sum_{t=1}^N \sigma_t^2 \right]^{1/2} \quad \dots, (1.32)$$

### **3. Applied Side**

#### **Introduction**

In this section, symmetric and asymmetric (nonlinear) GARCH modeling is applied to the energy market. We attempt to use the ARMA-GARCH family to model and to forecast the volatilities of Gasoline returns prices series in Erbil city under the different error distributions, and then compare the results and choose the appropriate model to forecast the volatility (conditional variance). We are going to use the sample of the historical daily Gasoline prices data spans over 8 years. The datasets will be analyzed using the results were extracted using econometrical software E-views version 9.

First, the data and its processing are described. Afterwards, by examining data set, it can be checked that there are serial correlation among observations of dataset and the volatility is not constant, so GARCH Models are appropriate. The best idea is to estimate ARMA-GARCH models in-sample periods and selection the best volatility models for the daily Gasoline prices data, depending on less value of (Akaike information criterion and Schwartz information criterion), also the parameters must be significant, in addition the residuals don't have the serial correlation and ARCH effect, as well as these models should have the higher value of log-likelihood. The effect of the random error type of models was also examined, by studying three types of statistical distributions (Normal, GED and Student-t). Finally, we evaluated out-of-sample forecasting performance of the volatility models, and then choose the best model to forecast the volatility of daily Gasoline prices returns data.

#### **3-1 Descriptive Statistics of the Data Set**

The data contain daily Gasoline prices time series. The data employed in this paper has been collected from the fuel stations (Qalat, Hoger, Yasameen, Akar and Shorsh) in Erbil city. This data consist of (2920) observations daily prices

on Gasoline covering the period 1/01/2010 to 31/12/2017. The sample is divided into two subsamples to permit more efficient model. The first subsample is called in-sample data set (Training sample) (seven years) starts from 1/01/2010 to 31/12/2016 with 2555 daily observations used to estimate the ARMA-GARCH models for underlying data and the second subsample is called out-sample data set (Testing sample) (one year) starts from 1/01/2017 to 31/12/2017 with 365 daily observations used to investigate the performance of volatility forecasting. The parameters of all the models are optimized on a training set; the testing set is used to compare quality of the models.

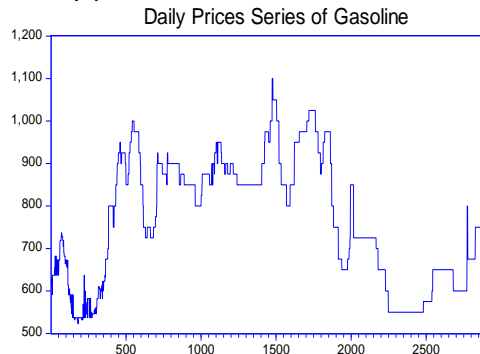
### **3-2 Time Series Analysis**

#### **3-2-1 Stationarity Study for Gasoline Prices Series Data**

Before the use of data to create the suitable model, the data needed to be tested for stationary to understand the nature of data. To study the stationary of the original daily Gasoline prices series we use the following:

##### **a) Time Plots of the Original Daily Gasoline Prices Series**

The first step of the analysis of any time series is to plot the data, based on the original observations to know the behavior and to see the visual structure of this data. Time series plot gives an initial clue about the nature of the series or shows an upward or downward trend, seasonal or cyclical fluctuation etc. Graphical representation suggests that the time series is stationary or not. We start by plotting the daily Gasoline prices series. Figure (3-1) shows the time series plot for original daily prices of Gasoline series.



**Figure (3-1): The Scatter Plots of the Daily Gasoline Price Series**

Figure (3.1) illustrates the original daily Gasoline prices series. The observed data show that there are periods with higher fluctuations, followed by periods with lower movements. The data appears non-stationary, with occasional jumps and spikes, i.e., its variance is changing with time, the volatility seems to change over time as well, indicating heteroscedasticity. But just looking at the time series graph is not enough to know how non-stationary the series is, so we have to use Ljung-Box test, correlogram and the unit root tests for data series.

**b) Ljung-Box Test and Correlogram for Original Daily Gasoline Prices Series**

Ljung-Box tests and correlogram for original daily Gasoline prices series given in [See Appendix No.1], we note from the table and correlogram of ACF and PACF the probabilities that corresponding to t-statistic less than ( $\alpha = 0.05$ ), in addition to autocorrelations coefficients approaching to one, this indicates that the original daily Gasoline prices series are non-stationary.

**c) Unit Root Tests for Original Daily Gasoline Price Series**

We are testing the original Gasoline price series for stationarity using the unit root tests of Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests to investigate whether the daily Gasoline price is stationary series. Table (3-2) gives results of unit root tests.

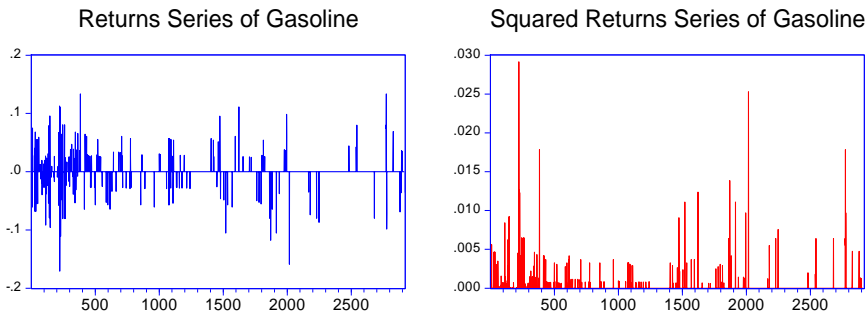
**Table (3-1):** Results of Unit Root Tests for Original Daily Gasoline Price Series

Null Hypothesis $H_0$ : Gasoline Price Series has a unit root (NonStationary)				
Test Statistic	Type of Model	5% Critical Value	Value of Test of Statistics	p-value
Augmented Dickey Fuller (ADF)	Intercept	-2.8623	-2.0635	0.2598
	Trend and Intercept	-3.4114	-2.3076	0.4291
	None	-1.9409	-0.1964	0.6156
Phillips-Perron (PP)	Intercept	-2.8623	-2.0021	0.2861
	Trend and Intercept	-3.4114	-2.2234	0.4757
	None	-1.9409	-0.2193	0.6074

According to above table, we see all the p-values of ADF and PP tests more than ( $\alpha = 0.05$ ) for daily Gasoline price series, then we not reject the null hypotheses  $H_0$ , this means that the daily Gasoline prices time series are non-stationary and time series data have a unit root has been justified. So we have to convert the data to returns series, to remove the effect of the mean and the variance of the time series using the transformation.

### 3-2-2 Transformation of Original Daily Gasoline Price Series to Returns

In order to adjust for a fair amount of the non-random effects, the returns of the daily time series is simply calculated from day to day. The currency Gasoline price series is transformed into daily log returns using the logarithm of the first difference, then the daily dataset is transformed into log-returns  $r_t$ , with  $y_t$  denoting the daily Gasoline price series observed at time  $t$ , by using the following equation:  $r_t = (\log y_t - \log y_{t-1})$ , which is presented in figure (3.2) and with squared log-returns series for the Gasoline price.



**Figure(3-2):** Graphic Representation of the Daily Log>Returns and Squared Log>Returns Series for the Gasoline Prices

The figure (3-2) shows that the mean returns are constant but the variances change over time around some normal level, with large (small) changes tending to be followed by large (small) changes of either sign, i.e. volatility tends to cluster. Periods of high volatility can be distinguished from low volatility periods. The presence of spikes and volatility clustering is quite obvious.

### 3-2-3 Summary Descriptive Statistics and Normality Tests of the Returns Series Data

Summary of the descriptive statistics and results of normality test (Jarque-Bera test) for the returns series of Gasoline prices data is presented in Table (3-2). The number of observations equals 2919. The mean and variance are all quite small.

**Table(3-2):** Descriptive Statistics and Normality Tests for Daily Gasoline Returns

Statistic	Gasoline Return Series	Statistic	Gasoline Return Series
Mean	4.49E-05	Skewness	-0.6381
Median	0.0000	Kurtosis	38.6315
Maximum	0.1335	Jarque-Bera	154613.5
Minimum	-0.1708	Probability-JB	0.0000
Std. Dev.	0.0146	Observations	2919

From above table, we notice that our dataset is extremely volatile. The data exhibits both positive and negative spikes / jumps. The mean and median of daily returns are not significantly different from zero. It suggests that returns Gasoline series in general decrease slightly overtime. The measures of skewness for the Gasoline returns series is -0.6381, there is not zero which means Gasoline returns series is asymmetric and skewed to the left(negatively skewed).On the other hand the returns series exhibit positive excess kurtosis, 38.6315. There is more than three, indicates the leptokurtic characteristic of the Gasoline daily returns distribution, which mean Gasoline returns have the fat-tail characteristic, greater peak at the mean than normal distribution, indicating the necessity of fat-tailed distribution to describe this variable, and these are some of the stylized facts observed in financial time series data. Based on the p-value of the Jarque-Bera tests, the p-value is less than 0.05, and then we reject the null hypothesis of normality at 5% for Gasoline returns series, so the distribution of the Gasoline returns is not normal distribution.

**3-2-4 the Ljung-Box Test for the Returns and Squared Returns of the Gasoline Series.**

Ljung-Box tests and correlograms for returns and squared returns of Gasoline Series are shown in the [Appendix No.2]. This test, which helps us to check whether the Gasoline returns, has serial correlation and the ARCH effects or not, the null hypotheses are the Gasoline returns don't have serial correlation

and ARCH effects, while the alternative hypothesis is opposite. Based on the assumption of 5% significance level, all of the p-values in the table and correlograms of ACF and PACF are smaller than 0.05, then we rejected the null hypothesis at 24th lag for Gasoline returns series which means the Gasoline returns have serial correlation and ARCH effect.

**3-2-5 Unit Root Test for Returns Series (Stationary)**

The unit root tests results for Gasoline returns series are shown in Table (3-3). This table displays the results of unit root tests using the ADF and PP tests at level with p-values and critical values for returns series of Gasoline prices. The null hypothesis of unit roots can be rejected to returns series at 5% level of significance.

**Table (3-3): Results of Unit Root Tests for Daily Gasoline Prices Returns Series**

Panel B: Unit Root Test of Gasoline Returns Series				
Null Hypothesis $H_0$ : Gasoline Returns Series has a unit root (Not Stationary)				
Test Statistic	Type of Model	5% Critical Value	Value of Test of Statistics	p-value
Augmented Dickey Fuller (ADF)	Intercept	-2.8623	-19.5220	0.0000
	Trend and Intercept	-3.4114	-19.5337	0.0000
	None	-1.9409	-19.5239	0.0000
Phillips-Perron (PP)	Intercept	-2.8623	-52.2072	0.0001
	Trend and Intercept	-3.4114	-52.2080	0.0000
	None	-1.9409	-52.2164	0.0001

According to the results in Table (3-3), we investigate the stationary of the returns series, the p-values are less than 5%. Therefore, we reject the null

hypothesis of “series has unit root” and conclude that the returns series is stationary. For this reason, we use returns series in the subsequent analysis.

**3-3 Construction Adequate Linear ARMA Models (Estimation Unconditional Mean Equation) for Daily Returns Series**

First step, we can construct suitable linear ARMA(p, q) models using the daily returns series of Gasoline prices because it is stationary at level5%. Using the Box-Jenkins modeling strategy using least squares method to estimate unconditional mean equation in the in-sample. Several ARMA models are fit to the returns series and the standardized residuals analyzed. By observing the autocorrelation (ACF) and partial autocorrelation functions (PACF), the rough p and q can be acquired, after comparing the model that gives us the lowest value of AIC and SIC selection criteria, and taking into account value of R<sup>2</sup>, also significant of parameters, the more accurate p and q will be picked up, to select the best fitted linear ARMA(p, q) model, by using different orders for Gasoline daily return series, chosen the optimal model among the candidate models after several attempts, taking into account ARCH effect and serial correlation. It was founded that the model ARMA(2,0) without a constant is the best model for Gasoline returns series. Table (3-4) and figure (3-3) observed the results of adequate estimated linear ARMA(2, 0)model and graph comparison among residuals actual and fitted series of ARMA(2, 0) Model.

**Table (3-4): Results of Adequate Estimated ARMA(p, q) Model Using Least Squares**

Method for Returns Series of the Gasoline Prices

Model	Coefficient	S.Error	t-statistic	Prob.	Log-Likelihood	A.I.C	SIC
ARMA(2,0)	-0.2215	0.0075	-29.6097	0.000	7203.924	-5.6397	-5.6351
SIGMASQ	0.0002	1.50E-06	138.7417	0.000			

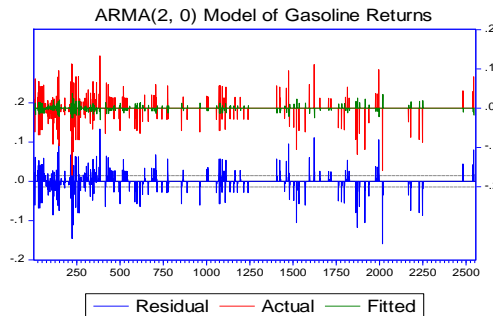
As shows in above table p-values of the parameters of ARMA(2,0) model are less than 0.05 significant level, that means the model is significant. Also show that the value of the log-likelihood for the estimated model was 7203.924 is very high value reflecting the efficiency of the model. Then the adequate estimated



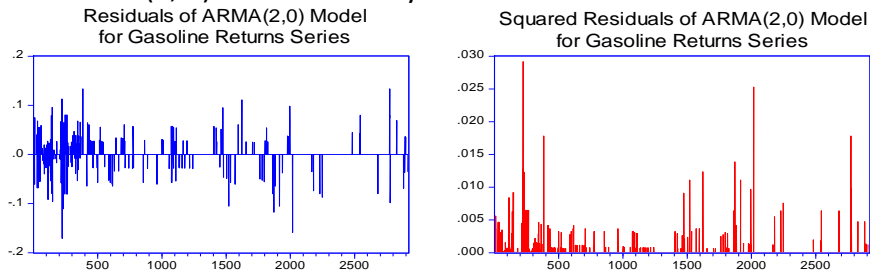
ARMA(p, q) model using least squares method for returns series of the Gasoline prices, given as follows;

$$\text{Gasoline Return } (r_t) = \text{ARMA}(2, 0) = - 2215 r_{t-2}$$

We plot the residual, squared residuals and fitted series derived from adequate ARMA(2, 0) model with actual series for daily returns of Gasoline prices series to compare among them, as below:



**Figure (3-3): Comparison among Residuals, Actual and Fitted Series of ARMA(2, 0) Model for Daily Gasoline Prices Returns Series**



**Figure(3-4): Residuals and Squared Residuals of ARMA(2, 0) Model**

According to figure (3-3) and (3-4) we see that there are periods of high volatility (big fluctuations) are followed by periods of high volatility and periods of low volatility (small fluctuations) trend to be followed by periods of low volatility of low volatility and etc. It seems that the residuals are stationary and volatility clustering. These suggest that residuals or error terms are conditionally heteroscedastic and when the residuals behaviors like this then us it can be represented by GARCH models, because the GARCH models is used for estimating volatility that takes care of volatility clustering issue.

**3-3-1 Residuals Diagnostics of ARMA(2,0) Model for Daily Returns of Gasoline Series.**

The diagnostics stage includes residuals analysis of estimated model. Now we want to test whether the heteroskedasticity (ARCH effect) and serial correlation problems are exist or not, with normality test for Jarque–Bera and that is permit using more formal Lagrange multiplier test for ARCH disturbances. Then we check the ACF and PACF of residuals and squared residuals. Table (3-5) and Appendix No.3 show results of residuals diagnostics of ARMA(2, 0) model for the daily returns of Gasoline series.

**Table (3-5):** Results of the ARCH-LM test, Ljung-Box test and Jarque-Bera Test on

Residuals of ARMA(2,0)Model for the Daily Returns of Gasoline Series

ARCH-LM Test Results $H_0$ : There is no ARCH Effect	
F-Statistic	44.3655
Prob. F(2,2549)	0.0000
<b>Obs*R-Squared</b>	85.8471
Prob. Chi-Square(2)	0.0000
Ljung-Box test of Standardized Residuals Test Results (24 Lags)	
$H_0$ : There is no Serial Correlation in the Residual	
Prob. of Q-Statistic	significant
Prob. of $Q^2$ -Statistic	significant
Jarque – Bera Test Resultfor Normality	
$H_0$ : The Residual hasNormal Distribution	
J-B Statistic	105521.8
Prob.	0.0000

All the p-values of tests statistics (F-statistic) and (Obs\*R-squared values: Chi-Square statistic) of ARCH-Lagrange multiplier (LM) test up to lag 2 for

residuals of ARMA(2,0) model for Gasoline returns are (0.000) less than 0.05 indicates the presence of ARCH effect in the residuals series of this model. Based on the results of Ljung-Box tests at 5% significance level, most of the p-values in the table and correlograms of ACF and PACF of residuals and squared residuals are smaller than 0.05, then we rejected the null hypotheses at 24th lag for residuals series which means the residual of ARMA(2,0) model for the Gasoline returns have serial correlation and ARCH effect. Also p-value of the Jarque-Bera test is less than 0.05, and then we reject the null hypothesis of normality at 5%, so the distribution of the residual of this model is not normal distribution, leptokurtic and the fat-tailed asymmetric distribution outperform the normal distribution, and un estimators are still consistent, and this model has two conditions, serial correlation and ARCH effect, therefore it should be appropriate to try modeling the volatility for Gasoline prices with the GARCH models.

### **3-4 Univariate Non Linear ARMA-GARCH Modeling for Daily Returns of Gasoline Prices**

After volatility clustering are confirmed with returns series and stationarity using ADF and PP tests, heteroscedasticity effects using ARCH-LM tests, and fitted adequate linear ARMA models using least squares method to estimate unconditional mean equations, the study focuses on determining the best fitted non-linear ARMA-GARCH models to the returns series, using maximum likelihood estimation method to estimate the conditional mean and variance equations of this model. Therefore, symmetric and asymmetric GARCH models are used for modeling the volatility in-sample dataset of daily returns for Gasoline prices series, under the different error distributions (Normal Distribution, General Error Distribution and Student-t Distribution). Then compare the results and choose the appropriate model that have lowest value of AIC and SIC selection criteria, moreover taking into consideration the parameters of the best selected model must be significant, there is no ARCH effect, no serial correlation, large value of Log-likelihood and residuals series are normal distribution. Then use these models to forecast the volatility (conditional variance).

The following tables and figures contain results of in-sample estimation of the important models, we obtained after hundreds of models have been tried, for purpose diagnostic the degree of effect in the model. Some of these models

had problems that didn't match all the assumptions. We are taken these models into consideration for the purpose of trade-offs between them.

**3-4-1 Results of Important Non Linear ARMA-GARCH Models for Gasoline Returns Series**

In order to capture the symmetries and asymmetries in the Gasoline returns series, five models have been used including; AR(2)-GARCH(1,1) model, AR(2)-GARCH-M(1,1) (Risk Premium: Standard Deviation) model, AR(2)-TGARCH(1,1,1) model, AR(2)-EGARCH-M(2,1,1) (Risk Premium: Variance) model and AR(2)-Power GARCH (2,1,1) model under different error terms distributions, to estimate conditional mean and conditional variance (volatility) in-sample dataset. All estimation results have been shown in the tables (3-9), (3-10) and (3-11) and figures (3-7) and (3-8).

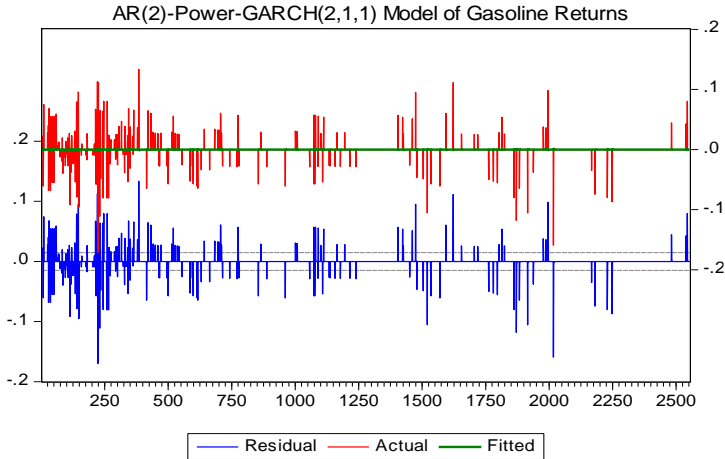
**Table (3-9): Estimation Results of Important Symmetric and Asymmetric Volatility Models for Returns of Gasoline**

Model	Significance of Parameters	Log-Likelihood	AIC	SIC
AR(2)-GARCH(1,1)-Norm	Significant	7506.031	-5.8747	-5.8655
AR(2)-GARCH(1,1)-Std.	Significant	11230.04	-8.7909	-8.7818
AR(2)-GARCH(1,1)-GED	Significant	7506.031	-5.8747	-5.8656
AR(2)-GARCH-M(1,1) (S.D)-Norm	Insignificant	7506.221	-5.8741	-5.8627
AR(2)-GARCH-M(1,1) (S.D)-Std.	Insignificant	11023.19	-8.6282	-8.6240
AR(2)-GARCH-M(1,1) (S.D)-GED	Significant	6397.213	-5.0056	-4.9942
AR(2)-TGARCH(1,1,1)-Norm	Significant	7438.819	-5.8213	-5.8099
AR(2)-TGARCH(1,1,1)-Std.	Significant	10268.41	-8.0371	-8.0257
AR(2)-TGARCH(1,1,1)-GED	Significant	8454.428	-6.6166	-6.6052
AR(2)-EGARCH-M(2,1,1) (var)-Norm	Insignificant	7563.430	-5.9173	-5.9013
AR(2)-EGARCH-M(2,1,1) (var)-Std.	Significant	16082.46	-12.5885	-12.5724
AR(2)-EGARCH-M(2,1,1) (var)-GED	Significant	6439.955	-5.0376	-5.0215
AR(2)-PGARCH-M(1,1,1)-Norm	Significant	7529.225	-5.8913	-4.2530
<b>AR(2)-PGARCH-M(2,1,1)-Std.</b>	<b>Significant</b>	<b>16333.25</b>	<b>-12.7849</b>	<b>-12.7790</b>
AR(2)-PGARCH-M(2,1,1)-GED	Insignificant	5438.033	-5.8776	-4.2369

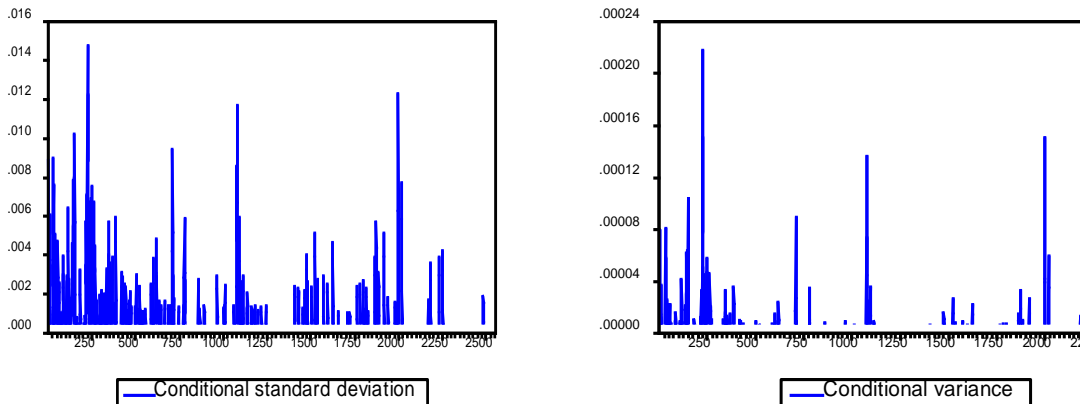
**Table (3-10): Estimation Results of the Best **Asymmetric (Non-Linear)** Volatility Model for **Gasoline** Returns**

Volatility Model	AR(2)-Power GARCH(2,1,1)		
	Student's t (d.f = 10)		
Coefficients of Mean Equation		Coefficients of Variance Equation	
AR(2)	<b>-0.0090</b>	w(Constant)	<b>0.0001</b>
z-Statistic	<b>-7.0296</b>	z-Statistic	<b>6.5808</b>
Prob.	<b>0.0000</b>	Prob.	<b>0.0000</b>
		$\alpha_1$ (ARCH effect)	<b>0.1916</b>
		z-Statistic	<b>34.7774</b>
		Prob.	<b>0.0000</b>
		$\alpha_2$ (ARCH effect)	<b>0.0747</b>
		z-Statistic	<b>26.3355</b>
		Prob.	<b>0.0000</b>
		$\beta_1$ (GARCH effect)	<b>0.4864</b>
		z-Statistic	<b>93.1377</b>
		Prob.	<b>0.0000</b>
		$\gamma$ (Leverage effect)	<b>0.0739</b>
		z-Statistic	<b>4.4041</b>
		Prob.	<b>0.0000</b>
Log-Likelihood	<b>16333.25</b>	$\delta$ (Power Term)	<b>0.5324</b>
AIC	<b>-12.7849</b>	z-Statistic	<b>51.8491</b>

SIC	<b>-12.7790</b>	Prob.	<b>0.0000</b>
-----	-----------------	-------	---------------



**Figure (3-7):** Comparison among Standardized Residuals, Actual and Fitted Series



**Figure (3-8):** Volatility process (Conditional Standard Deviation and Conditional Variance) Derived from the AR (2)-Power GARCH (2, 1,1) Standardized Residual

**Table (3-11)** Results of the Jarque-Bera, Ljung-Box and ARCH-LM tests on the Residuals of Important **Symmetric** and **Asymmetric** Volatility Models for Returns of **Gasoline** Price Series

	Error Distribution	Jarque – Bera Test for Normality		Ljung-Box Test <sup>a</sup>	ARCH-LM Test Results H <sub>0</sub> : There is no ARCH effect		
		J-B Statistic	P-value	P-value	F-Statistic	P-value F(1,2551)	Obs*R-Square
	Normal	140955.3	0.0000	insignificant	0.1484	0.7001	0.1485
	t	3975718	0.0000	insignificant	0.0670	0.7958	0.0670
	GED	140955.3	0.0000	insignificant	0.1484	0.7001	0.1485
(Std.Dev)	Normal	140859.7	0.0000	insignificant	0.1477	0.7008	0.1478
(Std.Dev)	t	42154737	0.0000	insignificant	0.0064	0.9362	0.0064
(Std.Dev)	GED	106149.9	0.0000	insignificant	0.0618	0.8037	0.0618
	Normal	180626.3	0.0000	insignificant	1.0816	0.2984	1.0820
	t	6473457	0.0000	insignificant	0.0413	0.8390	0.0413
	GED	256585.5	0.0000	insignificant	0.8764	0.3493	0.8768
-M(var)	Normal	159004.6	0.0000	insignificant	0.0361	0.8493	0.0361
-M(var)	t	373176.4	0.0000	insignificant	0.1973	0.6569	0.1975
-M(var)	GED	112528.4	0.0000	insignificant	0.3230	0.5699	0.3231
(1,1,1)	Normal	154336.6	0.0000	insignificant	0.2584	0.6112	0.2586
<b>(2,1,1)</b>	<b>t</b>	<b>1389708</b>	<b>0.0000</b>	<b>insignificant</b>	<b>0.1889</b>	<b>0.6639</b>	<b>0.1890</b>
(2,1,1)	GED	81381.89	0.0000	significant	0.3498	0.5543	0.3501

After comparing the results of the symmetric and asymmetric estimated models in the two tables (3-9) and (3-10) we found that the best model fits the volatility of Gasoline returns series is AR(2)-Power GARCH(1,1,1) non-linear asymmetric model with innovation student-t distribution (d.f=10), because all the coefficients of this model are statistically significant. In other words, the coefficients of conditional mean and variance equations, AR(2), constant ( $\omega$ ), ARCH term ( $\alpha_1$ ), ARCH term ( $\alpha_2$ ), GARCH term ( $\beta_1$ ), leverage term ( $\gamma$ ) and power parameter ( $\delta$ ) are highly significant at 5% level because (p-values < 0,05) and with expected sign. The significance of ( $\alpha_1$ ), ( $\alpha_2$ ), and ( $\beta_1$ ) indicates that two lagged squared disturbance and one lagged conditional variance have an impact on the conditional variance (today volatility), in other words this means that news (information) about volatility from the two previous periods has an explanatory power on current volatility. In the conditional variance equation, the estimated coefficient ( $\beta_1$ ) is greater than coefficients ( $\alpha_1$ ) and ( $\alpha_2$ ) which resembles that the market has a memory longer than two periods and that volatility is more sensitive to its lagged values than it is to new surprises in the market values. It implies that the shock of past volatility effect on current volatility. The sum of these coefficients ( $\alpha_1 + \alpha_2 + \beta_1$ ) is 0.7527, which infers that the shocks to the volatility will persist in the future periods. This implies that large changes in returns tend to be followed by large changes and small changes tend to be followed by small changes, which will therefore, confirm that volatility clustering is observed in Gasoline returns series. The ( $\gamma$ ) captures the asymmetric effect in the best fitted model. The coefficient of leverage effect ( $\gamma$ ), is positive and significant at 5% level, which gives the additional evidence of the volatility asymmetry, indicating that positive shocks (good news) are associated with higher volatility than negative shocks (bad news), The analysis reveals that there is a positive correlation between past returns and current volatility (leverage effect), hence AR(2)-Power GARCH(1,1,1) model supports for the presence of leverage effect on Gasoline returns series during the study period. Further, the appropriate model has large value of Log-likelihood and lowest values of AIC and SICS selection criteria.

In addition to, residual diagnostics checking for the best fitted model, according to table (3-11), ARCH-LM test is employed to check ARCH effect in



residuals and from the results of ARCH-LM test, it is inferred that the p-values >0.05, which led to conclude that the null hypothesis of ‘no arch effect’ is not rejected, which means there is no ARCH effect in the residuals of the model. Based on the results of Ljung-Box test at 5% significance level and Correlogram of ACF and PACF for squared standardized residuals Lags (24) of the best-fitting model [See Appendix No (4).], all the p-values in the table are more than 0.05 (insignificant), then we can’t reject the null hypothesis, which means there is no serial correlation in the residuals of AR(2)-Power GARCH(2,1,1) model. Also the p-value of the Jarque-Bera test is less than 0.05, and then we reject the null hypothesis of normality at 5%, so the distribution of the residuals is not normal distribution, but as estimators are still consistent, which implies that the variance equation is well specified for Gasoline returns series. Furthermore, comparison among standardized residuals, actual and fitted Series and conditional standard deviation and conditional variance derived from the best-fitting model as a measure of Gasoline price fluctuations, have been shown in the figures(3-7) and (3-8) respectively. The figure of volatility process shows that the volatility of Gasoline returns series have volatility clustering. Then the best estimated model which represents the volatility of Gasoline prices returns series is AR(2)-Power GARCH(2,1,1)-Std. non-linear asymmetric model, as follows;

$$\text{Gasoline Returns } \hat{r}_t = - 0.009 r_{t-2}$$

$$\hat{\sigma}_t^{0.5324} = 0.0001 + 0.1916 \left( \left| \varepsilon_{t-1} \right| - 0.0739 (\varepsilon_{t-1}) \right)^{0.5324} + 0.0747 \left( \left| \varepsilon_{t-2} \right| \right)^{0.5324} + 0.4864 (\sigma_{t-1}^{0.5324})$$

### 3-5 Forecasting Performance

One of the main objectives of this paper and time series analysis is to use the constructed model to forecast future values based on previously observed values of the series. The models were also evaluated in terms of their ability to forecast volatility of future returns for fuel prices. In this paper we use the out-of-sample forecast to investigate the forecasting performance. In this context, the measures of forecast evaluation used in the present paper include root mean square forecast error (RMSFE), mean absolute forecast error (MAFE), mean

absolute percent forecast error (MAPFE) and Theil’s inequality coefficient (TIC). These are used as relative measure to compare forecasts for the same series across different models, in order to acquire the appropriate model to forecast the volatility (conditional variance) we choose the model that has lowest values of forecast errors, and (TIC) less than one, which indicate best forecasting ability of volatility for the return series.

In order to acquire the appropriate model to forecast the volatility, and to see how the model might fit real data, we examine forecasts for out-of-sample data of the various important volatility models. The returns of Gasoline prices includes (2555) observations, seven years as in-sample dataset, which is used to estimate the parameters of the volatility models, and reserve the last year as out-of-sample dataset, including (365) observations, will be used to test the forecasting ability of the volatility models. Finally, we consider the in-sample and out-of-sample forecasting ability of the best adequate model for the returns of Gasoline prices, to compare between them, to find which one gives the best forecasting ability, we will show them in the later tables and figures.

**3-5-1TheOut-of-Sample Volatility Forecasts for Gasoline Prices Series**

Using results from the in-sample estimating, the AR(2)-Power GARCH(2,1,1) Std. model is selected as the representative asymmetric GARCH model in order to compare out-of-sample forecasting performance with implied volatilities and historical volatility. The results of the forecasting ability evaluation of the forecast models for the volatility of Gasoline returns series have been shown in the tables (3-17), (3-18) and figure (3-12).

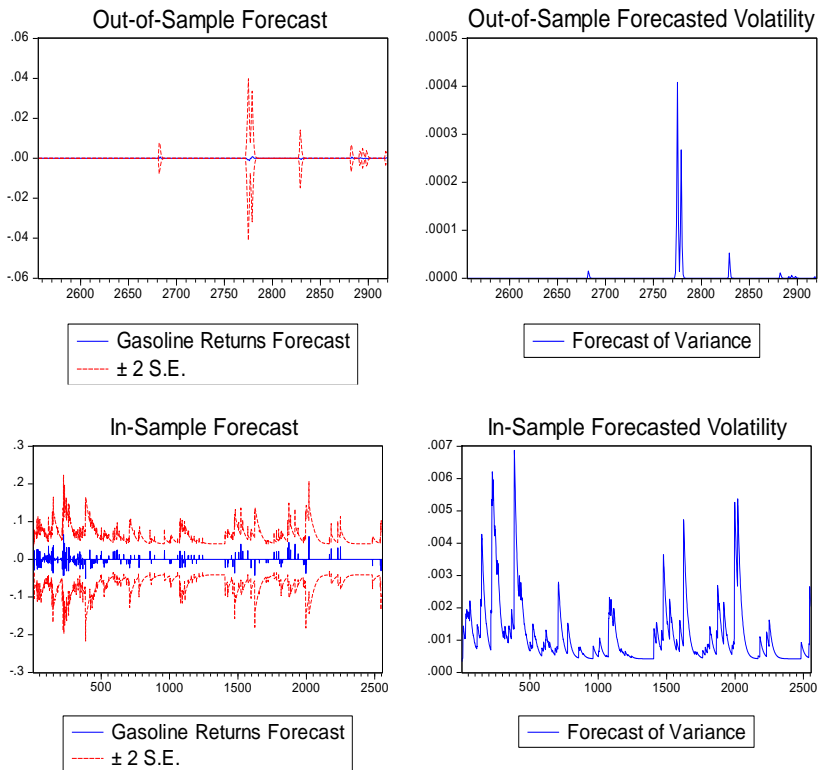
**Table (3-17):** Evaluation of Forecasting Power of the Forecast Model for the Volatility of **Gasoline** Prices Series

Model	RMSE	MAE	MAPE	Thiel Inequality Coefficient
AR(2)-GARCH(1,1)-Norm	0.013824	0.002581	3.578368	0.926847
AR(2)-GARCH(1,1)-Std.	0.013594	0.002399	3.565656	0.980245
AR(2)-GARCH(1,1)-GED	0.013824	0.002581	3.578368	0.926847

AR(2)-GARCH-M(1,1) (S.D)-Norm	0.013825	0.002723	3.578996	0.925758
AR(2)-GARCH-M(1,1) (S.D)-Std.	0.013568	0.002376	3.563884	0.988779
AR(2)-GARCH-M(1,1) (S.D)-GED	0.014920	0.005645	3.614585	0.806540
AR(2)-TGARCH(1,1,1)-Norm	0.013790	0.002557	3.576697	0.933141
AR(2)-TGARCH(1,1,1)-Std.	0.013693	0.002485	3.571656	0.953410
AR(2)-TGARCH(1,1,1)-GED	0.013585	0.002391	3.565089	0.982948
AR(2)-EGARCH-M(2,1,1) (var)-Norm	0.014124	0.002920	3.630067	0.923201
AR(2)-EGARCH-M(2,1,1) (var)-Std.	NA	NA	NA	NA
AR(2)-EGARCH-M(2,1,1) (var)-GED	0.014828	0.005254	3.616474	0.689708
AR(2)-PGARCH-M(1,1,1)-Norm	0.013823	0.002580	3.578301	0.927097
<b>AR(2)-PGARCH-M(2,1,1)-Std.</b>	<b>0.013557</b>	<b>0.002363</b>	<b>3.563124</b>	<b>0.992536</b>
AR(2)-PGARCH-M(2,1,1)-GED	0.015250	0.003239	3.624542	0.814566

Boldfaced number represents the minimal value in table.

Table (3-17) reports the forecast performance values for all the symmetric and asymmetric volatility models. The results indicate that the relative differences among forecasting performance measures are quite small for out-of-sample data. The forecasting results show after comparing the values of loss functions for all fifteen important volatility models, the lowest values of three evaluation statistics (RMSFE, MAFE and MAPFE) and the value of TIC is less than one, indicate that the AR(2)-Power GARCH(2,1,1)-Std. model is the most preferred among all the models in forecasting the volatility of Gasoline returns series, then this model has good forecasting power. Figure (3-12) presents the out-of- sample volatility forecast and variance forecast of the Gasoline returns. Thus the MA-Power GARCH model was found to be the best model to study the volatility behavior and the corresponding forecasting of returns.



**Figure (3-12):** The Out-of-Sample and In-Sample Volatility Forecasts for Gasoline Returns by Using AR(2)-Power GARCH(2, 1, 1) Model

**Table (3-18):** Comparison between Forecasting Performance In-Sample and Out-of-Sample for the Best Adequate Model of Gasoline Returns Series

Loss Function \ Sample	In-Sample Forecast	Out-of-Sample Forecast
RMSE	0.014615	0.013557
MAE	0.004241	0.002363
MAPE	7.373230	3.563124
Thiel Inequality Coefficient	0.714665	0.992536

According to table (3-18) We evaluated the forecasting ability of the AR(2)-Power GARCH(2,1,1) model with innovation t-distributions in the in-sample and out-of-sample for the volatility of Gasoline returns series. The results indicate that the relative differences among forecasting performance measures for both samples are quite small. Results obtained show that forecasting performance in the out-of-sample more accurate than forecasting performance in the in-sample.

## **4. Conclusions and Recommendations**

### **4-1 The Conclusions**

The main conclusions can be summarized as follows:

- 1.** The time series for original Gasoline prices is not stationary series, there is a general trend of ascending and descending, it has been converted into stationary returns series, using the logarithm of the first difference.
- 2.** The Gasoline daily returns series exhibits asymmetric and skewed to the left, positive the leptokurtic characteristic, which mean Gasoline returns have the fat-tail characteristic, the distribution is not normal distribution, and the presence of spikes and volatility clustering is quite obvious, and these are some of the stylized facts observed in financial time series data.
- 3.** By using least squares method to estimate unconditional mean equation in the in-sample. It was founded that the model ARMA(2,0) without a constant is the best model for Gasoline returns series.
- 4.** The research found that the AR(2)-Power GARCH (2, 1,1) model under student t distribution is best adequate model to estimate the volatility of Gasoline prices returns series, this means that news (information) about volatility from the two previous periods has an explanatory power on current volatility. Also In terms of the out-of-sample forecasting performance the results was conclusive. This volatility model is preferred based on the smallest values of three loss functions and TIC is less than one, indicate that the AR(2)-Power GARCH(2,1,1)-Std. model is the most preferred among all the models in forecasting the volatility of Gasoline returns series, then this model has good forecasting power.

5. The research found that the market has a memory longer than two periods and that volatility is more sensitive to its lagged values than it is to new surprises in the market values. It implies that the shock of past volatility has a persistent effect on current volatility.
6. The analysis reveals that there is a positive correlation between past returns and current volatility (leverage effect). The presence of leverage effects, giving the additional evidence of the volatility asymmetry, indicating that positive shocks (good news) are associated with higher volatility (conditional variance) than negative shocks (bad news) of Gasoline prices series.
7. We evaluated the forecasting ability of the AR(2)-Power GARCH(2,1,1) model with innovation t-distributions in the in-sample and out-of-sample for the volatility of Gasoline returns series. The results indicate that the relative differences among forecasting performance measures for both samples are quite small. Results obtained show that forecasting performance in the out-of-sample more accurate than forecasting performance in the in-sample.

#### **4-2 The Recommendations**

1. We recommend further research to forecast the volatility of Gasoline prices to include the examination of other GARCH families and using other types of distributions symmetric and asymmetric of random error for these models.
2. In future prospects the results of the case study can be used as a guide to generalize using GARCH family widely to model and forecast the volatility of other economic and financial variables in Kurdistan as a whole, such as the prices of; White Oil, Natural Gas, Benzene, Gold, exchange rate, price of electricity ... etc. Also, using multivariate GARCH models,
3. The study recommends using GARCH models in other various areas of interest in real life, which includes modeling and forecasting the volatility, for instance, environmental and pollution data, health researches in the context of longitudinal data, agriculture and geo- statistics.

#### **References**

- Andersen, T. G., et al. (2009). Handbook of financial time series, Springer Science & Business Media.
- Armstrong, J. S. (2001). Principles of forecasting: a handbook for researchers and practitioners, Springer Science & Business Media.
- Bollerslev, T. (1986). "Generalized autoregressive conditional heteroskedasticity." Journal of econometrics**31**(3): 307-327.
- Brockwell, P. J., et al. (2002). Introduction to time series and forecasting, Springer.
- Brooks, C. (2008). Introductory Econometric for Finance (Second Edi.), New York: Cambridge University Press.
- Cryer, J. D. and K.-S. Chan (2008). "Time series regression models." Time series analysis: with applications in R: 249-276.
- Ding, Z., et al. (1993). "A long memory property of stock market returns and a new model." Journal of empirical finance**1**(1): 83-106.
- Enders, W. (2015). "Applied Econometrics Time Series (Fourth Edi.)." Google Scholar.
- Engle, R. F. (1982). "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation." Econometrica: Journal of the Econometric Society: 987-1007.
- Engle, R. F., et al. (1987). "Estimating time varying risk premia in the term structure: The ARCH-M model." Econometrica: Journal of the Econometric Society: 391-407.
- Francq, C. and J.-M. Zakoian (2011). GARCH models: structure, statistical inference and financial applications, John Wiley & Sons.
- Franses, P. H. and D. Van Dijk (2000). Non-linear time series models in empirical finance, Cambridge University Press.
- Glosten, L. R., et al. (1993). "On the relation between the expected value and the volatility of the nominal excess return on stocks." The journal of finance**48**(5): 1779-1801.
- Gregoriou, G. N. (2009). Stock market volatility, CRC press.
- Kirchgässner, G., et al. (2012). Introduction to modern time series analysis, Springer Science & Business Media.
- Lütkepohl, H., et al. (2004). Applied time series econometrics, Cambridge university press.
- Montgomery, D. C., et al. (2015). Introduction to time series analysis and forecasting, John Wiley & Sons.
- Nelson, D. B. (1991). "Conditional heteroskedasticity in asset returns: A new approach." Econometrica: Journal of the Econometric Society: 347-370.
- Poon, S.-H. (2005). A practical guide to forecasting financial market volatility, John Wiley & Sons.
- Sahoo, P. "Department of Mathematics University of Louisville Louisville, KY 40292 USA."
- Satchell, S. and J. Knight (2011). Forecasting volatility in the financial markets, Elsevier.
- Shumway, R. H. and D. S. Stoffer (2000). "Time series analysis and its applications." Studies In Informatics And Control**9**(4): 375-376.

Tsay, R. (2002). Analysis of Financial Time Series. Financial Econometrics, A Wiley-Interscience Publication, John Wiley & Sons, INC, New York.

Wang, P. (2005). Financial econometrics, Routledge.

William, W. W. and W. Shyong (1994). Time series analysis, Addison-Wesley, Boston, MA, USA.

Kekalaki, E. and S. Degiannakis (2010). ARCH models for financial applications, John Wiley & Sons.

Zivot, E. and J. Wang (2006). "Modelling financial time series with S-PLUS." Springer: 429-478.

















































**Appendixes: Additional Figures and Tables**

















































**Appendix No.1: Results of Ljung-Box Test and Correlogram**  
**Correlogram of Gasoline Prices Series**

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 0.997	0.997	2905.9	0.000
		2 0.994	-0.033	5794.7	0.000
		3 0.991	0.101	8669.9	0.000
		4 0.989	-0.027	11531.	0.000
		5 0.986	0.021	14378.	0.000
		6 0.984	0.072	17214.	0.000
		7 0.982	0.006	20039.	0.000
		8 0.979	-0.112	22849.	0.000
		9 0.977	0.007	25644.	0.000
		10 0.974	0.037	28426.	0.000
		11 0.972	-0.013	31194.	0.000
		12 0.969	-0.041	33947.	0.000
		13 0.966	0.007	36686.	0.000
		14 0.963	-0.035	39411.	0.000
		15 0.960	-0.047	42117.	0.000
		16 0.957	-0.024	44807.	0.000
		17 0.954	0.000	47479.	0.000
		18 0.950	0.008	50135.	0.000
		19 0.947	-0.029	52773.	0.000
		20 0.944	0.013	55394.	0.000
		21 0.941	-0.018	57999.	0.000
		22 0.937	-0.061	60584.	0.000
		23 0.934	0.040	63151.	0.000
		24 0.930	0.024	65701.	0.000

**Appendix No.2: Results of Ljung-Box Test and Correlogram for Returns and Squared Returns of Gasoline Series**  
**Returns Series Returns Squared Series**



Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 0.028	0.028	2.2466	0.134
		2 0.233	0.232	160.98	0.000
		3 0.032	0.022	164.04	0.000
		4 0.061	0.006	175.03	0.000
		5 0.094	0.085	200.90	0.000
		6 -0.011	-0.032	201.25	0.000
		7 0.158	0.125	274.77	0.000
		8 -0.018	-0.019	275.76	0.000
		9 0.117	0.055	315.83	0.000
		10 -0.013	-0.018	316.30	0.000
		11 0.020	-0.023	317.42	0.000
		12 0.038	0.026	321.74	0.000
		13 -0.011	-0.007	322.07	0.000
		14 0.060	0.020	332.71	0.000
		15 -0.017	-0.004	333.53	0.000
		16 0.042	0.002	338.74	0.000
		17 -0.003	0.004	338.77	0.000
		18 0.005	-0.011	338.83	0.000
		19 0.046	0.040	344.97	0.000
		20 0.002	0.008	344.98	0.000
		21 0.100	0.071	374.58	0.000
		22 0.030	0.035	377.27	0.000
		23 0.053	0.003	385.44	0.000
		24 -0.003	-0.023	385.46	0.000

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 0.037	0.037	3.9635	0.046
		2 -0.181	-0.182	99.564	0.000
		3 0.025	0.041	101.37	0.000
		4 0.020	-0.017	102.50	0.000
		5 -0.113	-0.105	140.09	0.000
		6 -0.011	-0.000	140.41	0.000
		7 0.180	0.148	235.35	0.000
		8 -0.001	-0.015	235.35	0.000
		9 -0.097	-0.042	263.01	0.000
		10 0.027	0.015	265.10	0.000
		11 0.061	0.038	276.09	0.000
		12 -0.060	-0.026	286.49	0.000
		13 0.017	0.040	287.34	0.000
		14 0.113	0.063	324.99	0.000
		15 0.014	0.022	325.56	0.000
		16 -0.069	-0.017	339.53	0.000
		17 0.001	-0.005	339.54	0.000
		18 0.042	0.017	344.74	0.000
		19 -0.043	-0.018	350.26	0.000
		20 0.013	0.026	350.73	0.000
		21 0.112	0.068	387.36	0.000
		22 -0.040	-0.048	392.05	0.000
		23 -0.072	-0.015	407.39	0.000
		24 0.021	0.003	408.70	0.000

**Appendix No.3: Results of Ljung-Box Test and Correlogram for Residuals and Squared Residuals of ARMA(2,0) Model of Gasoline Returns Series.**  
**Residuals Squared Residuals**

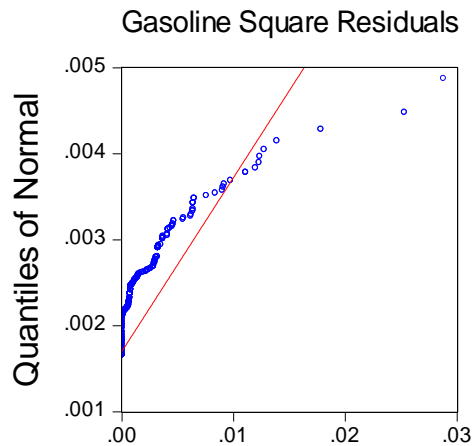
Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.007	0.007	0.1216	
		2	-0.000	-0.000	0.1216	0.727
		3	0.034	0.034	3.1326	0.209
		4	0.003	0.003	3.1561	0.368
		5	-0.054	-0.054	10.607	0.031
		6	0.010	0.009	10.842	0.055
		7	0.175	0.175	88.934	0.000
		8	0.005	0.006	88.992	0.000
		9	-0.061	-0.066	98.693	0.000
		10	0.018	0.004	99.536	0.000
		11	0.054	0.059	106.94	0.000
		12	-0.032	-0.010	109.59	0.000
		13	0.041	0.036	113.84	0.000
		14	0.108	0.071	144.09	0.000
		15	0.022	0.023	145.33	0.000
		16	-0.040	-0.020	149.42	0.000
		17	-0.006	-0.019	149.53	0.000
		18	0.039	0.023	153.41	0.000
		19	-0.024	-0.005	154.91	0.000
		20	0.019	0.013	155.81	0.000
		21	0.108	0.072	185.73	0.000
		22	-0.039	-0.048	189.74	0.000
		23	-0.057	-0.040	198.26	0.000
		24	0.001	-0.003	198.26	0.000

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	-0.003	-0.003	0.0282	0.867
		2	0.183	0.183	86.054	0.000
		3	0.003	0.004	86.079	0.000
		4	0.024	-0.010	87.540	0.000
		5	0.053	0.053	94.656	0.000
		6	-0.014	-0.017	95.162	0.000
		7	0.137	0.123	143.61	0.000
		8	-0.018	-0.013	144.46	0.000
		9	0.104	0.060	172.23	0.000
		10	-0.014	-0.010	172.71	0.000
		11	0.007	-0.024	172.83	0.000
		12	0.036	0.032	176.22	0.000
		13	-0.010	-0.004	176.50	0.000
		14	0.047	0.014	182.09	0.000
		15	-0.016	-0.006	182.72	0.000
		16	0.040	0.010	186.90	0.000
		17	-0.003	0.004	186.92	0.000
		18	0.009	-0.003	187.15	0.000
		19	0.040	0.033	191.17	0.000
		20	0.008	0.013	191.36	0.000
		21	0.088	0.065	211.18	0.000
		22	0.060	0.067	220.54	0.000
		23	0.041	0.006	224.97	0.000
		24	0.003	-0.019	224.99	0.000

**Appendix No.4:** Results of Ljung-Box Test, Correlogram and the Normal Quantile-Quantile Plots for Squared Residuals of ARMA(2,0)-Power GARCH(2,1,1)-(Std) Model of Daily Gasoline Returns Series.

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*	
		1	-0.009	-0.009	0.1893	0.664
		2	-0.009	-0.009	0.3788	0.827
		3	-0.009	-0.009	0.5685	0.904
		4	-0.009	-0.009	0.7585	0.944
		5	-0.009	-0.009	0.9487	0.967
		6	-0.009	-0.009	1.1390	0.980
		7	-0.009	-0.009	1.3296	0.988
		8	-0.009	-0.009	1.5195	0.992
		9	-0.007	-0.008	1.6463	0.996
		10	-0.008	-0.008	1.7975	0.998
		11	-0.009	-0.009	1.9888	0.999
		12	-0.008	-0.009	2.1403	0.999
		13	-0.001	-0.002	2.1424	1.000
		14	0.012	0.011	2.5084	1.000
		15	-0.009	-0.009	2.7008	1.000
		16	0.017	0.016	3.4449	1.000
		17	-0.002	-0.002	3.4566	1.000
		18	0.006	0.006	3.5463	1.000
		19	0.026	0.026	5.2478	0.999
		20	-0.007	-0.007	5.3820	1.000
		21	0.001	0.001	5.3837	1.000
		22	-0.009	-0.008	5.5724	1.000
		23	0.006	0.007	5.6745	1.000
		24	0.001	0.002	5.6767	1.000



Quantiles of Squared Residuals of ARMA(2,0)-PGARCH(2,1,1) Model

**پوخته**

ئامانجی توژیینه وه که بهراورکردنی به جی هیئانی ژماره یه که له مۆدیله کانی هه لگه راوه کان (نه بوونی جیگیری **GARCH** ) وه که یه که و وه که یه که نه بوون تاکی دووری له بونیائنانی مۆدیل و پیشبینی کردن به هه لگه راوه کانی نرخه کانی رۆژانه ی گازوایل له شاری هه ولیر. ئەم توژیینه وه مۆدیلی **GARCH** و **GARCH-M** و **TGARCH** و **EGARCH** و **PGARCH** هه لده بژیڕین بۆ شیکردنه وه ی داها تی رۆژانه ی گازوایل له گه ل لیکۆلینه وه ی کاریگه ری سێ جۆری جیاواز له دابه شکاروه کانی هه له ی هه ره مه کی ئه وانیش: دابه شکاروی نۆرمه لی و دابه شکاروی قۆتابی **t** و دابه شکاروی هه له ی گشتانن کراو، وه له دوایدا بهراورکردن له نیوان ئه نجامه کان و هه لبژاردنی مۆدیلی گونجاو بۆ پیشبینی کردن به هه لگه راوه کان. بژارده دابه ش کرا بۆ دوو بژارده ی به شی: ناو لیده نریت بژارده ی به شی یه که م به کۆمه لیک داتا له ناو بژارده (بژارده ی راهینان) به کارهینراو بۆ مه زنده کردنی مۆدیله کانی **ARMA-GARCH** بۆ داتای بنه ره تی، وه ناو لیده نریت بژارده ی به شی دووهم به کۆمه لیک داتای دهره وه ی بژارده (بژارده ی تاقیکردنه وه) به کارهینراو بۆ لیکۆلینه وه له به جی هیئانی پیشبینی کردن به هه لگه راوه کان. له ئه نجامی شیکردنه وه کان، گه یشتینه ئه نجام که باشتیرین مۆدیل که باشه بۆ هه لگه راوه کانی زنجیره ی داها تی کازوایل بریتیه له مۆدیلی وه که یه که نه بوون و هیللی نه بوونی **AR(2)-Power-GARCH(2,1,1)** وه کاتیک دابه ش ده بیت هه له ی هه ره مه کی مۆدیل به دابه شکاروی قوتابی **t** وه به پله ی سه ره خۆیی (10)، وه به جی هیئانی پیشبینی کراوی باشتیری هه بیت له مۆدیله کانی تر. ئەم ئه نجامه گرنگه بۆ چه ندين لایه نی دارایی وه که بریاره کانی وه به ره یئان و به نرخ کردنی هه بووه کان و تاییه تمه ندرکردنی هه گبه و کارگیری ترسناکی.

**ملخص****النمذجة والتنبؤ بالتقلبات الأسعار الكازولين باستخدام نماذج **GARCH******المتماثلة وغير المتماثلة في مدينة أربيل**

يهدف البحث إلى مقارنة أداء عدد من نماذج التقلبات (عدم الثبات) **GARCH** المتماثلة وغير المتماثلة احادي البعد في النمذجة والتنبؤ بالتقلبات الأسعار الكازولين اليومية في مدينة أربيل. يختار هذه

البحث نموذج GARCH و GARCH-M و TGARCH و EGARCH و PGARCH لتحليل العوائد اليومية للكازولين مع دراسة تأثير ثلاث انواع مختلفة من التوزيعات الخطأ العشوائي وهي: التوزيع الطبيعي والتوزيع الطالب-t والتوزيع الخطأ المعمم، ومن ثم المقارنة بين النتائج والاختبار النموذج المناسب للتنبؤ بالتقلبات. تم تقسيم العينة إلى عينتين الجزئية: تسمى العينة الجزئية الأولى بمجموعة البيانات داخل العينة (عينة التدريب) المستخدمة لتقدير نماذج ARMA-GARCH للبيانات الأساسية، وتسمى العينة الجزئية الثانية بمجموعة البيانات خارج العينة (عينة الاختبار) المستخدمة للتحقيق في أداء التنبؤ بالتقلبات. نتيجة للتحليلات، نستنتج أن أفضل نموذج يلائم التقلبات السلسلة العوائد الكازولين هو النموذج غير متمائل وغير خطي  $GARCH(2,1,1)$ -Power  $AR(2)$  وعندما يتوزع الخطأ العشوائي للنموذج توزيع الطالب t وبدرجة الحرية (10)، ولديه أداء التنبؤي أفضل من نماذج الأخرى. هذه النتيجة مهمة في العديد من المجالات المالية مثل القرارات الاستثمارية، وتسعير الأصول، وتخصيص الحقيبة وإدارة المخاطر.