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Analyzing the Distribution of Family Income in Erbil's Urban Population using Pareto Distribution and its Properties

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ABSTRACT

In the real life there are many phenomena that have Pareto distribution's properties. One of the useful applications of this distribution is family income. This study aims to understand Pareto distribution, its properties, and fitting the family income to Pareto distribution. The Data were collected through a questionnaire distributed to 290 families; the sample covered just urban population in Erbil city. The data analyses of the family income done through applying EasyFit v.5.4 mainly, Statgraphics v.15, and MS Excel 2016 programs. The Pareto distribution with two parameters (**α=0.646 and β=5**) was discovered to be the best fit for this type of data. It is suggested that local authorities adopt the proposed distribution to reap the benefits of applying pareto distribution to future KRG plans aimed at increasing the income of Kurdish families.

1. Introduction

Statistical distributions have many usages and advantages applied in any phenomenon or field in order to realize and estimate the basic characteristics of those variables belonging to that field. There are a large number of statistical distributions such as Binomial, Poisson, Gamma, Beta, Normal, Chi-square, Weibull, Pareto, etc.; each of them has its characteristics such as mean, variance, mode, median, and so forth. But the most crucial thing in these distributions is when and where it can be used or applied.

The purpose of this study is to present and discuss one of these distributions, the Pareto distribution, with application to family income in Erbil, because it is used to describe social, quality control, scientific, geophysical, and actuarial phenomena, among others. Furthermore, the emphasis will be on the verifying of essential properties of the Pareto distribution using some limited resources due to a shortage of resources related to this distribution.

The great importance of this study is to determine the distribution of family income in Erbil's urban population in order to better understand the nature of family income and plan for future actions. The general directorate of planning, the directorate of human development, the directorate of employment and social affairs, companies involved in product consumption, and so on will all benefit greatly from this information.

The following is a breakdown of the structure of this study. There are four sections: introduction, goal of study, literature review, and methods and materials of study. The practical study section is specified in section five which consists of the results including an overview of descriptive statistics and pareto distribution. The conclusions and recommendations section is specified in section six.

2. Study Objectives

The main objectives of this study that are related to Pareto distribution is using family income data in order to diagnose is it fitting Pareto distribution or not through some applying statistical software.

Depending on the above objectives, the main scientific hypothesis concerning this study states that the data of family incomes in Erbil city is distributed as Pareto random variable with 2 parameters (α, β).

3. Literature Review

After Pareto principles of his law concerning income and wealth, and then called Pareto distribution (80%/20%), many scientists and researchers worked with it and made great improvements in this field.

In 1931, Gibrat had made random growth models, the stochastic process is assumed to be scale independent and called it Gibrat’s law. In 1949, Zipf had many

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contributions in Pareto distribution for income. Champernowne is maybe the first of such random growth models for incomes (1953), and Simon and Bonini's for companies (1958). Kesten (1973), Gabaix (1999), and Luttmer (2007) are other examples of this approach. Many researchers had used Pareto distributions as primitives' distribution, such as Lucas (1978), Helpman et al. (2004), Chaney (2008), Terviö (2008) and Gabaix and Landier (2008). A study of Song et al. (2015) on the determinants of wage inequality across and within firms (Geerolf, 2017) (B. B. Aghevli, 12 Mar 2012) (Amita Majumder, 8 August 2006). Additionally, two researchers employed pareto models for top income in 2019, where they explored several sorts of bias that might occur in empirical investigations and offered some practical suggestions (Charpentier & Flachaire, 2019). The recent empirical work was for Abdul Majid et al. (2021) concerning the composite Pareto distributions for modelling family income distribution in Malaysia. They tried to composite Pareto models that fitted to the Malaysian family income data of several years.

4. Methods and Materials

4.1. Methodology

Since this study trying to understand Pareto distribution and its properties, fit the income family to Pareto distribution, and data had been collected through interviewing, the mixed research of pure and applied research is being taken into consideration. The Pareto Distribution has been used to fit the collected data.

4.2. Pareto Distribution

The creator of Pareto distribution was an Italian scientist called Vilfredo Federico Domaso Pareto who was born on July 15th, 1848 and died on August 19th, 1923. Pareto was raised in a family with a good income, enjoying the many benefits that accrued to people of his class during that age. He had a suitable education in both France and Italy. In the secondary schools Pareto studied mathematical and classical subjects basically. In 1864, he graduated from high school and enrolled in Turin university-Italy. Pareto's first study was published in 1866 and got his first degree in mathematics and physical sciences in 1867. Directly he enrolled in the engineering school at the same university and had graduated in 1870. Pareto worked as an engineer, politician, technician, economist, and sociologist. After 1890, Pareto started his scientific career at university of Lausanne and published remarkable books. His original appointment was as an associate professor in April 1893; exactly one year later he became a full professor (Kleiber & Kotz, 2003).

Pareto's two main contributions to economics, which also contain his developments in income distributions, are "Cours d'e´conomie politique" in 1897 and "Manuale di economia politica" in 1906. His massive contribution to sociology was the four-volume The Mind and Society. His research "Cours" contained the so-called Pareto's law of income distribution which states shortly that the distribution of income and wealth in any country and in all ages follows an invariant pattern. He remarked that around 80% of the land in Italy was owned by 20% of the population. Later, Pareto observed that income and wealth distribution among nations followed a similar distribution (Parmenter, 2007).

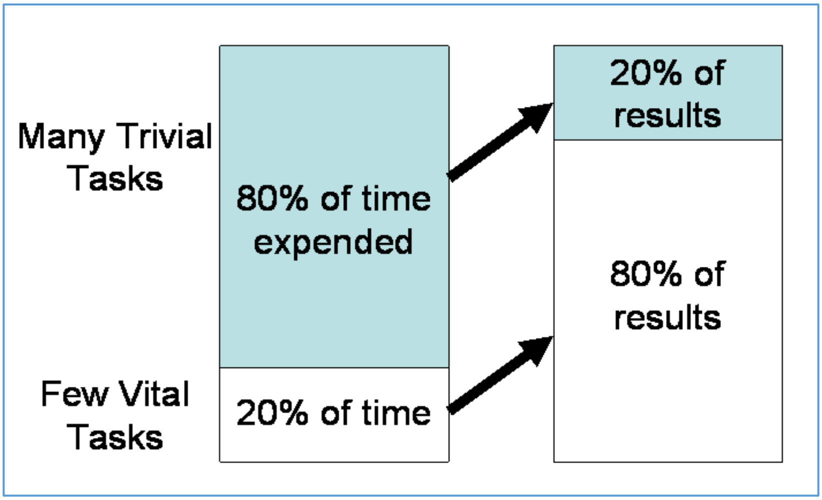

Figure (1): The rule of Pareto's distribution (Anon., n.d.)

Figure 1 shows the rule of 80/20 clearly, and the followings are some examples of 80/20 Pareto's rule:

- ❖ 80% of automobile traffic jams arises from 20% of the sites or roads.
- ❖ 80% of the profits comes from 20% of business.

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- 20% of efforts give 80% of results.
- 80% of peas in a garden were produced from 20% of the pea pods.
- 20% of patients make 80% of the clinic center issues.
- 20% of a products defects cause 80% of its issues.

4.2.1. Pareto distribution Properties

A random variable X is said to have a Pareto distribution with two parameters α and β, (α>0,β >0) if the pdf of X is:

$$f(x;\propto,\beta) = \begin{cases} \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}} & \beta < x < \infty \\ 0 & O.W \end{cases}$$

Pareto distribution has another probability distribution function when β=1, and it can be written in the following form:

$$f(x;\propto) = \begin{cases} \frac{\propto}{x^{\alpha+1}} & X > 1 \\ 0 & O.W \end{cases}$$

where:

α: is the shape parameter (minimum possible value of x)

β: is the scale parameter

It has another pdf which is called sometimes Lomax distribution. It can be written in the following form:

$$f(x;\propto,\beta) = \begin{cases} \frac{\alpha\beta^{\alpha}}{(x+\beta)^{\alpha+1}} & 0 < x < \infty \\ 0 & O.W \end{cases}$$

Below are the most important properties of the Pareto distribution.

***1*- Prove of the C.D.F of Pareto distribution:** (Mark Maxwell, 2015) (Kleiber & Kotz, 2003)

$$\mathrm{F(x)} = \mathrm{p(X \le x)} = \int_{\beta}^{x} f(\chi;\alpha,\beta)\,\mathrm{dx}$$

$$\mathrm{F(x)} = \alpha\beta^{\alpha}\int_{\beta}^{x}\frac{1}{x^{\alpha+1}}dx$$

$$= \alpha\beta^{\alpha}\int_{\beta}^{x} x^{-\alpha-1}dx$$

$$= \alpha\beta^{\alpha}\left[\frac{x^{-\alpha}}{-\alpha}\right]_{\beta}^{x}$$

$$= \beta^{\alpha}\left[\frac{-1}{x^{\alpha}}\right]_{\beta}^{x} \Rightarrow \mathrm{F(x)} = \beta^{\alpha}\left[\frac{-1}{x^{\alpha}} - \frac{-1}{\beta^{\alpha}}\right]$$

$$\mathrm{F(x)} = \beta^{\alpha}\left[\frac{1}{\beta^{\alpha}} - \frac{1}{x^{\alpha}}\right]$$

$$\mathrm{F(x)} = \frac{\beta^{\alpha}}{\beta^{\alpha}} - \frac{\beta^{\alpha}}{x^{\alpha}}$$

$$\mathrm{F(x)} = 1 - \frac{\beta^{\alpha}}{x^{\alpha}}$$

$$\mathrm{F(x)} = 1 - \left(\frac{\beta}{x}\right)^{\alpha}$$

$$\therefore \mathrm{F(x)} = \begin{cases} 0, & x < \beta \\ 1-\left(\frac{\beta}{x}\right)^{\alpha}, & \beta < x < \infty \\ 1, & x \to \infty \end{cases}$$

<u>2-The mean of Pareto distribution:</u> (atthew J. Hassett, 2006)

$$f(x;\alpha,\beta) = \alpha\beta^{\alpha}x^{-\alpha-1} \qquad X>\beta \ , \quad \alpha, \beta>0$$

$$E(x) = \int_{\beta}^{\infty} x f(x;\alpha,\beta)dx$$

$$E(x) = \alpha\beta^{\alpha}\int_{\beta}^{\infty} x x^{-\alpha-1}dx$$

$$E(x) = \alpha\beta^{\alpha}\int_{\beta}^{\infty} x^{-\alpha}dx \Rightarrow E(x) = \frac{\alpha\beta^{\alpha}}{1-\alpha}\left[x^{1-\alpha}\right]_{\beta}^{\infty}$$

$$E(x) = \frac{\alpha\beta^{\alpha}}{1-\alpha}\left[\frac{1}{x^{\alpha-1}}\right]_{\beta}^{\infty} \Rightarrow E(x) = \frac{\alpha\beta^{\alpha}}{1-\alpha}\left[\frac{1}{(\infty)^{\alpha-1}} - \frac{1}{\beta^{\alpha-1}}\right]$$

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$$E(x)=\frac{\alpha\beta^{\alpha}}{1-\alpha}\left[0-\frac{\beta}{\beta^{\alpha}}\right]\Rightarrow E(x)=\frac{\alpha\beta^{\alpha}}{1-\alpha}*\frac{-\beta}{\beta^{\alpha}}$$

$$E(x)=\frac{-\alpha\beta}{1-\alpha}\Rightarrow E(x)==\frac{\alpha\beta}{\alpha-1}$$

3- The variance of Pareto distribution: (atthew J. Hassett, 2006)

Var(x) = $E(x)^2-[E(x)]^2$

$$E(x)^2=\int_{\beta}^{\infty}x^2 f(\chi;\alpha,\beta)dx$$

$$=\int_{\beta}^{\infty}x^2\alpha\beta^{\alpha}x^{-\alpha-1}dx$$

$$=\alpha\beta^{\alpha}\int_{\beta}^{\infty}x^{1-\alpha}dx$$

$$=\alpha\beta^{\alpha}\left[\frac{x^{2-\alpha}}{2-\alpha}\right]_{\beta}^{\infty}\Rightarrow E(x)^2\frac{\alpha\beta^{\alpha}}{2-\alpha}\left[\frac{1}{(\infty)^{\alpha-2}}-\frac{1}{\beta^{\alpha-2}}\right]$$

$$=\frac{\alpha\beta^{\alpha}}{2-\alpha}\left[0-\frac{\beta^2}{\beta^{\alpha}}\right]=\frac{-\alpha\beta^2}{2-\alpha}\quad *(-1)$$

$$=\frac{\alpha\beta^2}{\alpha-2}$$

$$Var(x)=\frac{\alpha\beta^2}{\alpha-2}-\left[\frac{\alpha\beta}{\alpha-1}\right]^2$$

$$=\frac{\alpha\beta^2}{(\alpha-2)}-\frac{\alpha^2\beta^2}{(\alpha-1)^2}\Rightarrow Var(x)=\frac{\alpha\beta^2(\alpha-1)-\alpha^2\beta^2(\alpha-2)}{(\alpha-2)(\alpha-1)^2}$$

$$=\frac{\alpha\beta^2(\alpha^2-2\alpha+1)-\alpha^2\beta^2(\alpha-2)}{(\alpha-2)(\alpha-1)^2}\Rightarrow Var(x)=\frac{\alpha^3\beta^2-2\alpha^2\beta^2+\alpha\beta^2-\alpha^3\beta^2+2\alpha^2\beta^2}{(\alpha-2)(\alpha-1)^2}$$

$$\therefore \text{Var(x)}=\frac{\alpha\beta^2}{(\alpha-2)(\alpha-1)^2}, where\, \alpha>2$$

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4-The median of Pareto distribution: (Arnold, 2015)

$$\int_{\beta}^{me} f(x;\alpha,\beta)\,\mathrm{dx} = \frac{1}{2}$$

$$\int_{\beta}^{me} \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}\,\mathrm{dx} = \frac{1}{2}$$

$$\propto \beta^{\alpha} \int_{\beta}^{me} x^{-\propto-1}\mathrm{dx} = \frac{1}{2}$$

$$\propto \beta^{\alpha}\left(\frac{x^{-\propto}}{-\propto}\Big|_{\beta}^{me}\right) = \frac{1}{2}$$

$$-\beta^{\alpha}(me^{-\alpha} - \beta^{-\alpha}) = \frac{1}{2}$$

$$1 - \frac{\beta^{\alpha}}{me^{\alpha}} = \frac{1}{2} \rightarrow \frac{\beta^{\alpha}}{me^{\alpha}} = \frac{1}{2}$$

$$me^{\propto} = 2\beta^{\alpha}$$

$$\therefore me = \beta(2)^{\frac{1}{\propto}} \qquad x > \beta$$

5-The moment about origin of one parameter: (atthew J. Hassett, 2006) (Arnold, 2015)

$$E(x^{r}) = \int_{1}^{\infty} x^{r}\frac{\propto}{x^{\propto+1}}\,dx$$

$$= \propto \int_{1}^{\infty} \frac{1}{x^{\propto-r+1}}\,dx$$

$$let \propto_{new} = \propto - r$$

$$= \propto \int_{1}^{\infty} \frac{1}{x^{\propto_{new}+1}}\,dx$$

$$= \propto \int_{1}^{\infty} x^{-\propto_{new}-1}\,dx$$

$$= \frac{\propto}{\propto_{new}}\left[x^{-\propto_{new}}\right]_{1}^{\infty} = \frac{\propto}{\propto_{new}}$$

$$\therefore\ E(x^{r}) = \frac{\propto}{\propto - r} \qquad r < \propto$$

6-Skeweness: (atthew J. Hassett, 2006)

$$\textbf{Skeweness} = \sqrt{\frac{\alpha-2}{\alpha}}\frac{2(\alpha+1)}{\alpha-3}$$

7-Kurtosis: (Arnold, 2015)

$$\textbf{Kurtosis} = \frac{6(\alpha^3+\alpha^2-6\alpha-2)}{\alpha(\alpha-3)(\alpha-4)}$$

10. Unbiased Estimator: (Vonta, et al., 2008)

$$\mathrm{E(x)} = \frac{\alpha\beta}{\alpha-1}$$

$$f(x;\alpha,\beta) = \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}$$

$$E(\bar{x}) = E\left(\frac{\sum xi}{n}\right) = \frac{\sum E(xi)}{n} = \frac{n\,\frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}}{n} = \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}$$

Maximum likelihood estimator of pareto distribution: (Arnold, 2015)

$$f(x;\lambda) = \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}$$

$$\mathrm{L}(\propto,\beta) = \frac{\alpha^{n}\beta^{n\alpha}}{\prod_{i=1}^{n} {x_i}^{\alpha+1}}$$

$$\ln l(\alpha,\beta) = n\, ln\alpha + n\,\alpha \ln\beta - (\alpha+1)\sum_{i=1}^{n}\ln x_i$$

$$\frac{\partial\ \mathrm{Ln}\ l(\alpha,\beta)}{\partial\alpha} = \frac{n}{\alpha} + n\, ln\beta - \sum_{i=1}^{n}\ln x_i - 0 = 0$$

$$\frac{n}{\alpha} = \sum_{i=1}^{n}\ln x_i - n\ln\beta$$

$$\hat{\alpha} = \frac{n}{\sum_{i=1}^{n}\ln x_i - n\ln\beta}\ , \qquad \beta = 1$$

$$\hat{\alpha}_{m.l.e} = \frac{1}{\frac{\sum_{i=1}^{n} \ln x_i}{n}} = \frac{1}{\ln \bar{x}}$$

$$\frac{\partial^2 \text{Ln } l(\propto, \beta)}{\partial \alpha^2} = -\frac{-2n}{\alpha^2} < 0$$

$$\therefore \ \hat{\alpha} = \frac{1}{\ln \bar{x}} is\ m.l.e$$

$$\frac{\partial \text{ Ln } l(\alpha, \beta)}{\partial \beta} = 0 + \frac{n\alpha}{\beta} - 0$$

$$\frac{n\alpha}{\beta} \neq 0$$

∴ we can solve it Logically \\

$x_1, x_2, \dots, x_n$ is a r.v. of size n with Pareto (β< x)

$let\ y_1 = \min\{x_1, x_2, \dots, x_n\} = x_{min}$

$let\ y_n = \max\{x_1, x_2, \dots, x_n\} = x_{max}$

$\beta < y_1 < y_2, \dots < y_n$

$\therefore \hat{\beta}_{m.l.e} = y_1$

The Pareto distribution has a relationship with exponential distribution, therefore, if X distributed Pareto and $Y = Ln(\frac{X}{\beta})$, then Y distributed exponential with parameter α (Vonta, et al., 2008).

5. Results and Discussion

5.1. Data Collection

The applied data in this study was from the author's collection of data on family income insufficiency for Erbil city, where the study population was Urban population in Erbil city. The actual sample size used in the analysis was 290 due to missing, misleading data or incomplete interviews. The sample covered just urban population in Erbil city.

The data entering and analyzing were done using EasyFit software V.5.4, and Microsoft Excel 2013. The author tried to use other software packages like SPSS, Statgraphics for their flexibility and usage but EasyFit has more flexibility than others. This section will be divided into 2 main sub-sections related to Pareto distribution for the Erbil's family income in 2015. The obtained data were manipulated, processed,

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and analyzed through EasyFit, Statgraphics, and Microsoft Excel 2013. Depending on the hypotheses that were referenced in the previous sections, some tests are had been used in this section to fit the data to Pareto distribution such as Kolmogorov-Smirnov, Chi-Square, and Anderson-Darling tests for goodness of fit test that determine the how well the theoretic distribution fits tentative distribution..

5.2. Descriptive Statistics

Table 1 presents the major statistics regarding Erbil's family income in 2015. This table shows that the lowest income is about 100 thousand IQD and the highest income is around 100 million IQD, it means a big range. Also, the median is one million IQD less than the mean which is equal to 1.7 million IQD, i.e. that the distribution of income has right tail and not symmetric.

Table (1): Summary statistics of family income in Erbil's city (2015)

Statistic	Values		Standard Error		
N	*292*				
Range	*99899500*				
Minimum	*100500*				
Maximum	*100000000*				
Sum	*524215000*				
Mean	*1795256.85*		*344626.19*		
Std. Deviation	*5888974.84*				
Variance	*34680024626241.60*				
Skewness	*16.07*		*.14*		
Kurtosis	*268.29*		*.28*		
Mode	*1000000*				
Percentiles	***10th***	***25th***	***50th (Med.)***	***75th***	***90th***
	500300	*750000*	*1000000*	*1787500*	*3000000*

Concerning the descriptive statistics, the most important thing is Pareto chart for this article. Figure 2 illustrates that citizens who located in 2 groups of family income class is (900,000–1,100,000 IQD) and (More than 2,100,000 IQD) have most incomes.

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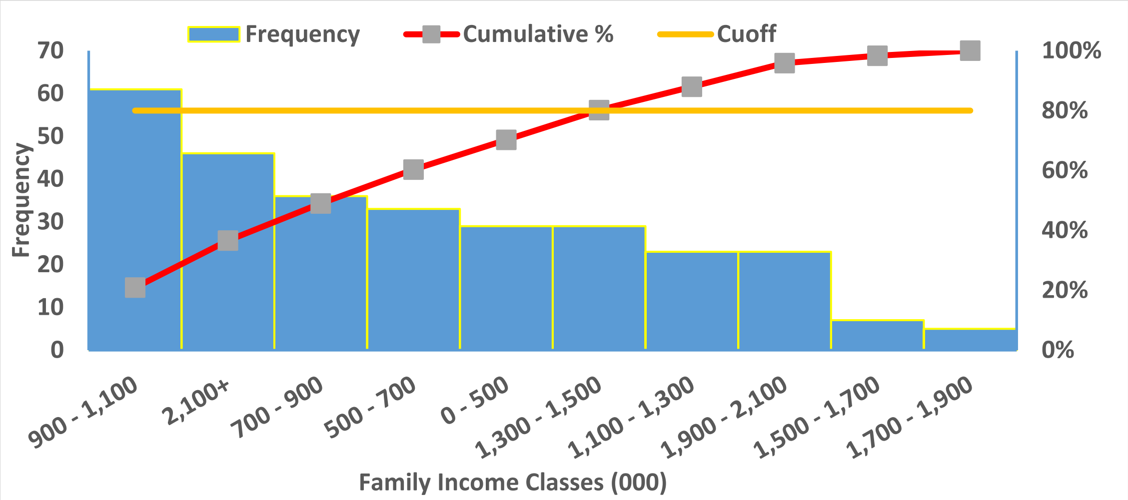

Figure (2): Pareto chart for family income of Erbil city in 2015

Approximately 50% of the total income is with the people in income class with more than 2,100,000 IQD and approximately 30% of total income in the hands of people in income group 900,000 – 1,100,000 IQD. Therefore, without taking any tests for fitting the family income data to Pareto distribution, it is clear that the data is distributed Pareto as presented in table 2. However, goodness of fit tests must be taken in order to be more confident.

Table (2): Frequency distribution and percentages of total income for Erbil's families

Classes (000)	Frequency	Relative frequency	Total Income Per Class	% of Total Income
2,100+	*46*	*15.8%*	*262780000*	*50.1%*
900 - 1,100	*61*	*20.9%*	*61306000*	*11.7%*
1,900 - 2,100	*23*	*7.9%*	*46000000*	*8.8%*
1,300 - 1,500	*29*	*9.9%*	*42981000*	*8.2%*
700 - 900	*36*	*12.3%*	*29241000*	*5.6%*
1,100 - 1,300	*23*	*7.9%*	*27736500*	*5.3%*
500 - 700	*33*	*11.3%*	*21330000*	*4.1%*
0 - 500	*29*	*9.9%*	*12140500*	*2.3%*
1,500 - 1,700	*7*	*2.4%*	*11650000*	*2.2%*
1,700 - 1,900	*5*	*1.7%*	*9050000*	*1.7%*

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Total	***292***	***100%***	***524215000***	***100.0%***

5.3. Fitting of Family Income to Pareto Distribution

This section is the most critical section in whole study because its results will lead to have conclusions about the study hypotheses. Therefore, accurate result of this section will be a basis for right conclusions and recommendations.

Using EasyFit and Statgraphics, table 3 describes the goodness of fit tests through Kolmogrov-Smirnov test and Anderson-Darling test. Therefore, it obvious that the null hypothesis cannot be rejected at a significance level equal to 0.01 (α=0.01) for both tests. Concerning χ^2 test, it cannot be used because there the estimated values are less than 5.

The estimated parameter for Pareto distribution are α=0.646 (shape parameter) and β=5 (scale parameter). Then the probability density function (PDF) is as follows:

$$f(x) = \begin{Bmatrix} \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}} & \beta < x < \infty \\ 0 & o.w \end{Bmatrix} = \begin{Bmatrix} \frac{0.646(5)^{0.646}}{x^{1.646}} & \beta < x < \infty \\ 0 & o.w \end{Bmatrix}$$

Table (3): Goodness of fit tests for family income in Erbil's city

Kolmogorov-Smirnov					
Sample Size Statistic P-Value	10 Groups 0.42672 0.03614				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.3226	0.36866	0.40925	0.45662	0.48893
Reject?	Yes	Yes	Yes	No	No
Anderson-Darling					
Sample Size Statistic	10 Groups 3.1968				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	No	No

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6. Conclusions and Recommendations

Depending on the results of the analyses in section five, the most important conclusion is that the family income in Erbil city distributed Pareto with 2 parameters (a=0.646 and b=5) and 20% of the groups earning approximately 80% of the total family incomes for the selected sample.

Therefore, depending on the study results and conclusions, it is recommended to use Pareto distribution for family incomes of all governorates of Kurdistan region in order to understand people's attitudes and preparing future improving plans for better life and increasing incomes.

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شیکردنەوەی دابەشی داهاتی خێزان لە کۆمەڵگای شارستانی هەولێر بە بەکارهێنانی دابەشی پاریتۆ و تایبەتمەندییەکانی

پـوختـە:

لە ژیانی ڕاستەقینەدا زۆر دیاردە هەن کە تایبەتمەندیەکانی دابەشکردنی پاریتۆیان هەیە. یەکێک لە کارە بەسوودەکانی ئەم دابەشە داهاتی خەڵکە. ئەم توێژینەوەیە ئامانجی تێگەیشتنە لە دابەشکردنی پاریتۆ، تایبەتمەندیەکانی، و گونجاندنی داهاتی خێزان بۆ دابەشی پاریتۆ. ئەم داتایە کۆکراوەتەوە لە ڕێگای فۆڕمس ڕاپرسی کە دابەشکراوە لەسەر 290 خێزاندا، بژاردەکە تەنها دانیشتوانی شارستانی هەولێریان بەخۆ گرتووە. شیکردنەوەی داتاکانی داهاتی خێزان کە لە ڕێگای بەکارهێنانی پڕۆگرامەکانی EasyFit v.5.4 بە شێوەیەکی سەرەکی، ستاتگرافیک v.15، و MS Excel 2016 ئەنجام دراوە. لە دەرئەنجام دا دەرکەوت کە دابەشی گونجاو بۆ ئەم جۆرە داتایانە دابەشی پاریتۆ بوو بە 2 زانراو ($\alpha=0.646$ و $\beta=5$). پێشنیار دەکرێت دابەشی پێشنیارکراو بەکاربهێنرێت لە لایەن دەستەلاتی ناوخۆ بۆ ئەوەی سوود لەو توێژینەوە ببینرێت بۆ پلانەکانی داهاتووی حکومەتی هەرێمی کوردستان بۆ ئەوەی داهاتی خێزانە کوردەکان باشتر بکرێت.

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تحليل توزيع دخل الأسرة لسكان المناطق الحضرية في أربيل باستخدام توزيع باريتو وخصائصه

الملخص:
في الحياة الواقعية، هناك العديد من الظواهر التي لها خصائص توزيع باريتو، أحد التطبيقات المفيدة لهذا التوزيع هو دخل المواطنين. تهدف هذه الدراسة إلى فهم توزيع باريتو وخصائصه وملائمة دخل الاسر لتوزيع باريتو. لقد تم جمع البيانات من خلال توزيع إستمارة الاستبانة على 290 أسرة، حيپ شملت العينة سكان الحضر فقط في مدينة أربيل. وتم تحليل بيانات دخل الأسرة من خلال تطبيق برنامج EasyFit (v.5.4) بشكل رئيسي، Statgraphics v.15، و MS Excel 2016. تم اكتشاف أن التوزيع المناسب لهذا النوع من البيانات كان توزيع باريتو وبمعلمتين (α = 0.646 و β = 5). يوصى باستخدام التوزيع المقترح وتتبناها السلطات المحلية لجني فوائد تطبيق توزيع باريتو على خطط حكومة إقليم كردستان المستقبلية الهادفة إلى زيادة دخل الاسر الكوردية.