



Using of Multivariate Hotelling T^2 Quality Control Charts in Bazyan Cement Factory

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ABSTRACT

Cement is representing the major element of concrete ingredient and most of Kurdish houses and buildings depending on concrete structure. Cement has physical and chemical characteristics that rely on them to attain good quality. Using quality control charts to know that the process is under control or not could be apply on cement factories in Kurdistan Regional Government (KRG). The objective of this paper is to use univariate and multivariate Quality Control Charts (QCC) to monitor the cement production in Bazyan factory. The data analyses were done through Statgraphics v.16 mainly for 4 cement characteristics (1 physical and 3 chemical) with 100 observations for each characteristic. The data was divided into 25 subgroups and then X-bar charts founded for the variables and compared them with the results of multivariate QCC using Hotelling T^2 . It was discovered that the process is not under control due to 2 chemical characteristics (Al_2O_3 and SO_3). It is recommended to use the suggested multivariate chart by responsible authority especially the directorates concerned with monitoring of cement factories.



1. Introduction

The basic components of concrete is cement. Concrete is formed via mixing cement, water, sand, and rock. Manufacturing of cement is controlled through chemical combination of calcium, silicon, aluminum, iron and other elements. Cement factory laboratories check each step in the manufacture of cement by frequent chemical and physical tests. The laboratories are analyzing and testing the completed product to ensure that it complies with all industry characteristics (Anon., n.d.).

Kurdistan mostly is relying on cement for construction of buildings and houses. Therefore, it is very important to have some factories for cement productions. One of the famous factories in KRG (Kurdistan Regional Government) is Bazyan factory for cement production. The government must have a monitoring and assessment of those production because making any mistakes with the physical and chemical cement elements will lead to bad quality of the production. So, the role of statistics in this situation will be very significant and could help the related governmental directorates to determine the out of control production process through quality control charts (QCC). Meaning that there are some defective parts in the process of production must be fixed or the process will lead to defective production. A process that is out of control indicates the existence of non-random variation generated by identifiable, particular causes, referred to as assignable causes. These assignable causes result in the process being uncontrollable or statistically unstable.

The great importance of this study is to private or governmental institutions who have direct or indirect relationship with cement and concrete such as Ministry of industry of KRG, private companies especially Bazyan cement factory, directorate of construction laboratory, directorate of quality control, and etc. In another hand, using of multivariate methods to test the quality of production is better than univariate methods because studying production ingredients together will reveal and determine problems more concisely.

The remaining of this article is organized as follows. The second section is related to objective of the study, methods and materials of the study is the address of third section, fourth section is concerning the results which consists of an overview of descriptive statistics and quality control mainly Univariate Quality Control Charts

(UQCC) and Multivariate Quality Control Charts (MQCC), and the last section is devoted for conclusions and recommendations. Each section may be divided into sub-sections depending on the subjects and requirements of the study.

2. Study Objectives

The main objective of this study is to find the UQCC and MQCC through some statistical tools such as \bar{X} chart, standard deviation chart, or Hotelling T^2 chart for chemical and physical properties of cements in Bazyan factory for cement production in 2016. The other objective is comparing the UQCC and MQCC in order to have the total information about cement ingredients altogether.

Depending on the above objectives, the authors stated main scientific hypotheses concerning this study and they are as follows:

The physical and chemical properties of cement production in Bazyan factory are under control, i.e. each point on the chart is in-control range and falls within the upper and lower bounds.

3. Methods and Materials

3.1 Data Collection

The data that used in this study is belonging to the directorate of construction laboratory in Erbil governorate. The data is concerning the Bazyan cement factory for a period starting from January, 2016 until December, 2016. This historical data is related to some physical and chemical properties of cement of 100 samples.

The data entering and analyzing were done using Statgraphics V16 and SPSS V.23 in order to obtain the descriptive and inferential analyses about 4 variables of cement properties, one is physical property which is called compressive strength property of cement after 7 days, and 3 chemical properties that are called Aluminum oxide (Al_2O_3), Sulfur trioxide (SO_3), Tricalcium Silicate (C_3S).

3.2 Methodology

Since this study trying to understand the behaviors of cement properties, finding the QCC, and data had been collected from historical sources of KRG ministry of industry/construction laboratory, the applied research (Quantitative Research) is

being taken into consideration. The UQCC and MQCC have been used to analyze the physical and chemical cement properties because it is one of the most important components of concrete.

3.3 Quality Control Charts (QCC)

The story of QCC have been used to inspect the quality of production when the Physicist Walter A. Shewhart introduced the QCC concept in the Bell Telephone Laboratories approximately one century ago (Montgomery, 2009). In 1931 he published a book related to QCC and started with univariate variables. However, after him many improvements had occurred especially with MQCC in many fields such as health, trade, agriculture, and so forth. The general purpose for QCC is to provide information that can be used to uncover systematic patterns by comparing expected variance versus observed variance. When talking about environment of production in QCC perspective, it means improving product quality and productivity in order to maximize a company's profits (Deming, 1982).

3.3.1 Univariate Quality Control Charts (UQCC)

The UQCC is a graphical display of a one variable quality characteristic that has been computed from a sample versus the sample number or time. The chart is consisting of three main elements: center line (CL) or baseline or target line that represents the total mean of the quality properties corresponding to the in-control state, Lower horizontal line, called the lower control limit (LCL), and upper horizontal line which is called upper control limit (UCL), as shown in figure 1 (Gibb & Harrison, 2010). These control limits are selected, so that if the process is in-control, nearly all of the sample points will fall between the two lines. As long as the points plot within the control limits, the process is assumed to be in-control, and no action is necessary. Setting UCL and LCL bounds, a procedure is to set them 3σ away from the center line, which signifies three standard deviations away from the arithmetic mean. The value of standard deviation can be determined by sample observations of X variable. The following formulas is used for setting the limits 3σ (Montgomery, 2009):

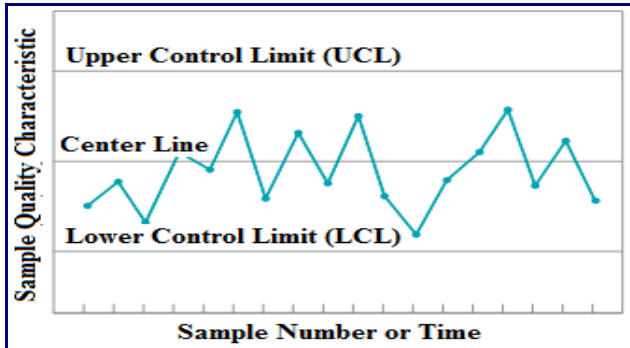


Figure (1): Typical UQCC

$$\begin{aligned}
 LCL &= \bar{\bar{X}} + 3\sigma \\
 CL &= \bar{\bar{X}} \quad \dots (1) \\
 LCL &= \bar{\bar{X}} - 3\sigma
 \end{aligned}$$

where $\bar{\bar{X}}$ is the total average, and σ is the standard deviation of the average.

There are many other UQCCs that could be used to control the production line. The most important Statistical Process Control (SPC) charts are as follows (Oakland, 2008):

- i.) $\bar{\bar{X}}$ chart as described in equation 1;
- ii.) Range chart;
- iii.) S chart;
- iv.) $\bar{\bar{X}}$ and S chart;
- v.) $\bar{\bar{X}}$ and R chart;
- vi.) Observation chart;
- vii.) P chart;
- viii.) Cumulative Sum (CUSUM) chart and other kind of charts;

There are 6 basic factors that should be checked for QCC, and if any one of these standards is met, then the process will be out of control (Oakland, 2008) (Schafer, et al., 2011).

1. One or more points are outside of the UCL and LCL.
2. At least seven consecutive points are on the same side of the center line.
3. Two out of three consecutive points are outside a 2σ line.
4. Four out of five consecutive points are outside of a 1σ line.

5. Any pattern that is noticeable that is non-random or is systematic in anyway.
6. One or more points are close to the upper or lower control limits.

3.3.2 Multivariate Quality Control Charts (MQCC)

The information inside multivariate data are more than a univariate data. At the same time, accounting for deviation in several variables requires both an overall measure of deviation of the observation from the targets in addition to having information about the correlation combinations of data (**Fuchs & Kenett, 1998**). Except UQCC, another technique will be used in this paper for analyzing MQCC through Hotelling T^2 statistic. This statistic is an analog of the X-bar (\bar{X}) Shewhart chart (**Montgomery, 2009**). There are other technics such as Multivariate Exponential Moving Averages (MEWMA) control chart but it is outside the research goals and objectives.

3.3.2.1 Hotelling T^2 Control Chart

This type of control charts considering that there are more than one quality characteristics x_1, x_2, \dots, x_p are jointly distributed according to the multivariate normal distribution (x_1, x_2, \dots, x_p are correlated). Let $\mu_1, \mu_2, \dots, \mu_p$ be the mean values of the quality characteristics, and let $\sigma_1, \sigma_2, \dots, \sigma_p$ be the variances of x_1, x_2, \dots, x_p , respectively. The covariance among x_1, x_2, \dots, x_p is denoted by a $p \times p$ matrix Σ (Montgomery, 2009). Therefore, the multivariate normal probability density function is as follows (Morrison, 1976):

$$f(\underline{x}) = \frac{1}{(2\pi)^p |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\underline{x} - \underline{\mu})' \Sigma^{-1}(\underline{x} - \underline{\mu})\right\} \quad \dots (2)$$

$$-\infty \leq \underline{x} \leq \infty \quad , \quad -\infty \leq \underline{\mu} \leq \infty \quad \text{and} \quad \Sigma \geq 0$$

Where: $\underline{x}' = (x_1, x_2, \dots, x_p)$, $\underline{\mu}' = (\mu_1, \mu_2, \dots, \mu_p)$ and

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1p} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \dots & \sigma_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \sigma_{p3} & \dots & \sigma_p^2 \end{bmatrix}$$

Assuming that $\sigma_1^2, \sigma_2^2, \dots, \sigma_p^2, \sigma_{12}, \sigma_{13}, \dots, \sigma_{p1}, \dots$ are unknown and that m such samples are available. The sample means, variances and covariance are calculated from each sample as follows (Montgomery, 2009) (Fuchs & Kenett, 1998):

$$\bar{x}_{jk} = \frac{1}{n} \sum_{i=1}^n x_{ijk} \quad \left\{ \begin{array}{l} j = 1, 2, \dots, p \\ k = 1, 2, \dots, m \end{array} \right\} \dots (3)$$

$$S_{jk}^2 = \frac{1}{n-1} \sum_{i=1}^n (x_{ijk} - \bar{x}_{jk})^2 \dots (4)$$

Where x_{ijk} is the i^{th} observation on the j^{th} quality characteristic in the k^{th} sample. The covariance between quality characteristic j and quality characteristic h in the k^{th} sample is as follows (Montgomery, 2009):

$$S_{jhk} = \frac{1}{n-1} \sum_{i=1}^n (x_{ijk} - \bar{x}_{jk})(x_{ihk} - \bar{x}_{hk}) \quad \{j \neq h\} \dots (5)$$

Then, the statistics $\bar{x}_{jk}, S_{jk}^2, S_{jhk}$ in equations 3, 4, and 5 are averaged over all m samples in order to have the followings:

$$\bar{\bar{x}}_j = \frac{1}{m} \sum_{k=1}^m \bar{x}_{jk} \dots (6)$$

$$\bar{S}_j^2 = \frac{1}{m} \sum_{k=1}^m S_{jk}^2 \dots (7)$$

$$\bar{S}_{jh} = \frac{1}{m} \sum_{k=1}^m S_{jhk} \dots (8)$$

The $\bar{\bar{x}}_j$ are the elements of the vector $\bar{\bar{\mathbf{X}}}$, and the $p \times p$ average of sample covariance matrices \mathbf{S} is formed as follows:

$$\mathbf{S} = \begin{bmatrix} \bar{S}_1^2 & \bar{S}_{12} & \bar{S}_{13} & \dots & \bar{S}_{1p} \\ \bar{S}_{21} & \bar{S}_2^2 & \bar{S}_{23} & \dots & \bar{S}_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{S}_{p1} & \bar{S}_{p2} & \bar{S}_{p3} & \dots & \bar{S}_p^2 \end{bmatrix} \dots (9)$$

The test statistic for each univariate sample means is:

$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \dots (10)$$

By squaring the two sides of the above equation, it will be:

$$t^2 = \frac{(\bar{x} - \mu)^2}{S^2/n} = n(\bar{x} - \mu)(S^2)^{-1}(\bar{x} - \mu) \dots (11)$$

Using the matrices and for $(k = 1, 2, \dots, m)$, the results will be as follows:

$$T_k^2 = n(\underline{\bar{x}}_k - \bar{\bar{x}})' S^{-1}(\underline{\bar{x}}_k - \bar{\bar{x}}) \dots (12)$$

The average of the sample means $\bar{\bar{X}}$ and covariance matrices \mathbf{S} is an unbiased estimate of $\underline{\mu}$ and $\underline{\Sigma}$, respectively when the process is in-control. In this form, the procedure is usually called the Hotelling T_k^2 control chart. This is a directionally invariant control chart; that is, its ability to detect a shift in the mean vector only depends on the magnitude of the shift, and not in its direction. So the points plotted on the chart representing the T_k^2 values, which represents the vertical axis of the chart (Wang, 2012).

The (construction stage) is to obtain an in-control set of observations so that control Limits can be established for which the monitoring of future production control is limits (Montgomery, 2009):

$$UCL = T^2 \sim \frac{2(m+1)(n-1)}{m(n-1)-1} F_{\alpha, p, mn-m-p+1} \dots (13)$$

$$LCL = 0$$

4. Results

This section will be divided into 2 main sub-sections related to data analysis for the Bazyan cement factory. The obtained data were analyzed through Statgraphics, and SPSS. Depending on the hypotheses that were referenced in the second section (physical and chemical properties of cement production in Bazyan factory are under

control), UQCC and MQCC approaches had been used in this section to analysis and finding the quality control patterns in the data.

4.1 Univariate Quality Control Charts (UQCC)

In the UQCC, the researcher will find the QCC for each of the four variables separately in order to know are they under control or not. Table 1 presents the most important elements of X-bar chart including UCL, LCL, and CL for all the variables. The data is divided into 25 groups, each of which contains four elements, i.e. the data, which has 100 elements for each variable, was divided into 25 groups, each of which contains four elements. The rationale for separating the data into 25 groups is because statistical literature suggests that the number of groups could not be fewer than 25.

Table (1): X-bar chart elements for four cement characteristics in Bazyan factory

Variable names	Center Line (CL)	UCL	LCL	Mean	SD
Physical characteristic: Compressive strength	41.65	48.68	34.62	41.65	4.69
Chemical characteristic: Al ₂ O ₃	5.73	7.57	3.88	5.73	1.23
Chemical characteristic: SO ₃	3.50	3.95	3.04	3.50	0.30
Chemical characteristic: C ₃ S	19.84	24.08	15.60	19.84	2.83

Now, let us see the X-bar control charts for the nominated variables. Figure 2 shows that all the control charts are out of control except physical characteristic of cement which is compressive strength but even this characteristic has a problem with the pattern. So, we cannot say that the process is under control.

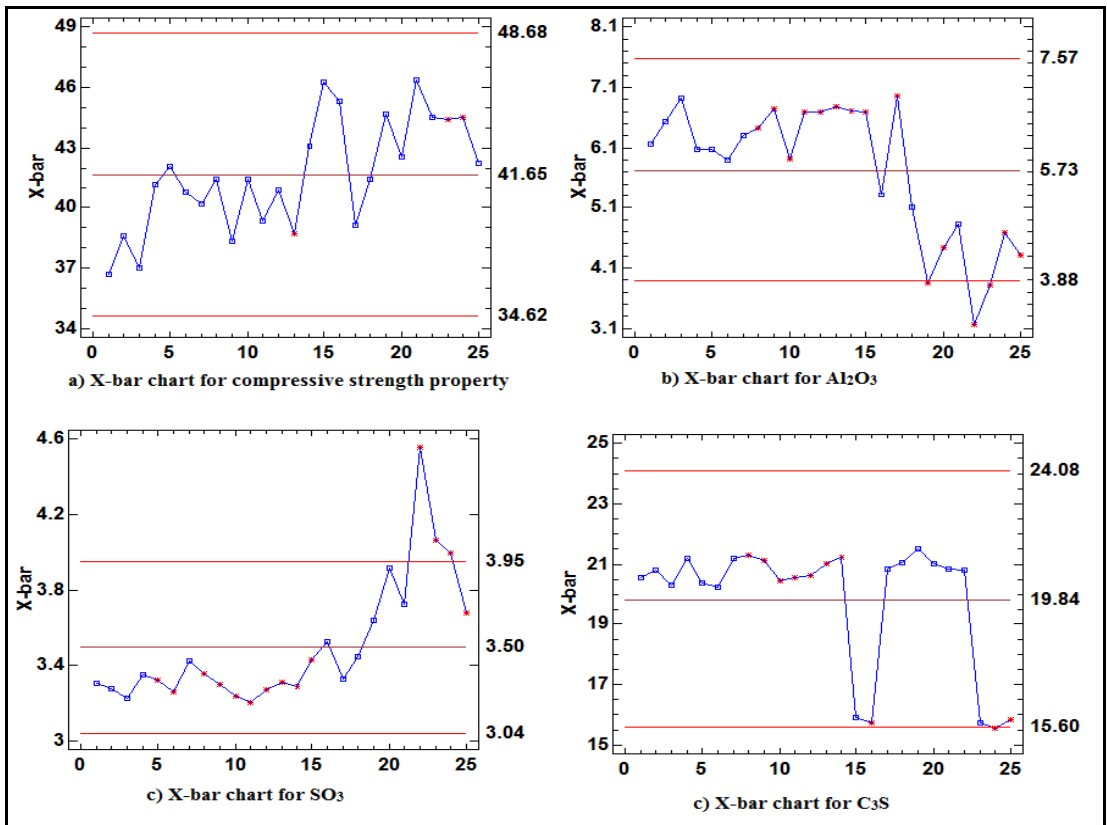


Figure (2): UQCC for cement characteristics: a) Compressive Strength, b) Al₂O₃, c) SO₃, d) C₃S

4.2. Multivariate Quality Control Charts (MQCC)

This section is the most significant section for the whole study because its results will lead to have conclusions about the study hypotheses of controlling the process depending on all variables. The averages of each group are presented in appendix. It is worth to say that all 4 variables are correlated significantly to each other with $\alpha=0.05$, and could be use MQCC using Hotelling T^2 statistic. For finding the T^2 statistic (eq. 9), the covariance matrix is illustrated in table 2.

Table (2): Covariance matrix for the cement characteristics in Bazyan factory

Variable names	Compressive strength	Al ₂ O ₃	SO ₃	C ₃ S
Compressive strength	27.05	-3.17	1.14	-9.82
Al ₂ O ₃	-3.17	2.63	-0.59	3.70

SO ₃	1.14	-0.59	0.23	-0.87
C ₃ S	-9.82	3.70	-0.88	21.20

Figure 3 shows the MQCC constructed for the 4 data variables. Unlike most control charts which treat variables separately, this procedure takes into account possible correlations between the variables. It is assumed that the variables follow a multivariate normal distribution. It seems from figure 3 that the process of cement production depending on 4 characteristics is out of control.

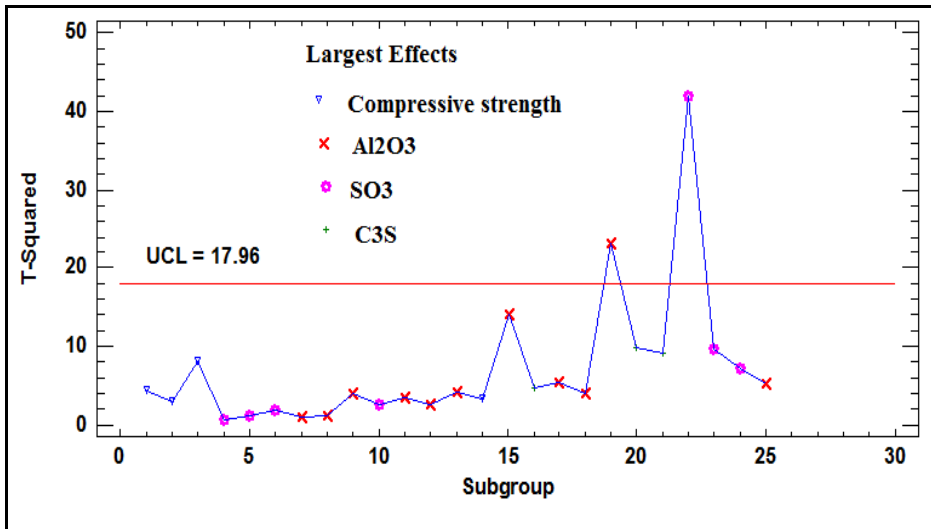


Figure (3): MQCC for cement characteristics of Bazyan factory

The control limits have been placed so as to give a 0.27% false alarm rate. For the T^2 chart, 2 cases were above the control limit. Since the probability of seeing 2 or more points above the control limit just by chance is 0.00209844 if the data come from the assumed multivariate normal distribution, we can declare the process to be out of control at the 95% confidence level.

Comparing UQCC from figure 2 and MQCC from figure 3, it is clear that chemical ingredients (SO₃ and Al₂O₃) are the causes of which the process is not under control in figure 3. However, in figure 2, the reason of out of controlling are 3 chemical characteristics. So, there is a confident evidence that using of MQCC show more information than UQCC, as presented in control Ellipsoid chart below.

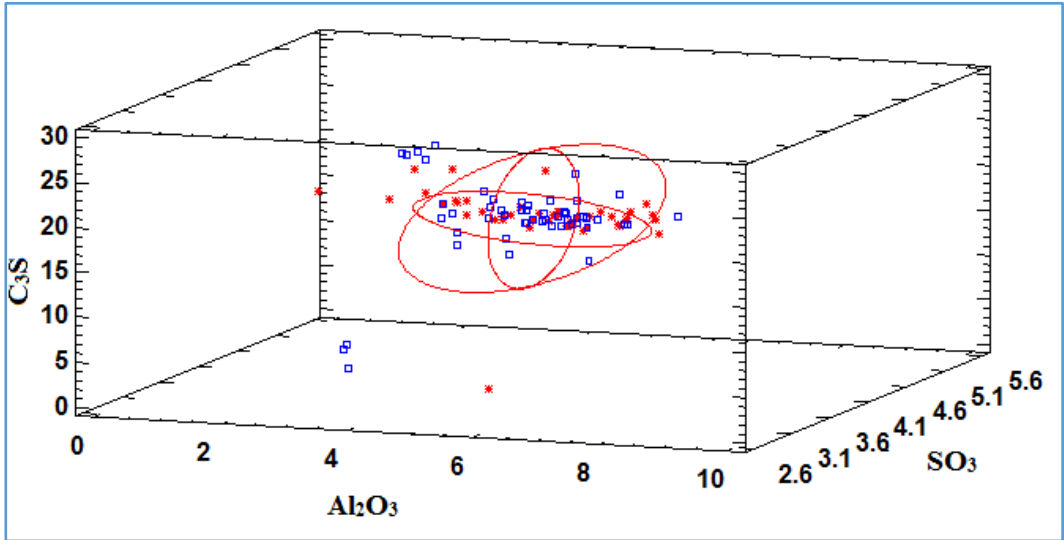


Figure (4): A 3D control Ellipsoid of MQCC for cement characteristics of Bazyan factory

5. Conclusions

Depending on the results of the analyses in section four, the most important conclusions are as follows:

The results showed that the process of cement production is not under control using UQCC and MQCC for Bazyan factory and the main reasons are chemical characteristics of Al_2O_3 and SO_3 . Also, it appears that it will be better for the factory to use MQCC than UQCC because it describes more patterns and real production issues.

Therefore, depending on the study results and conclusions, it is recommended to use the proposed QCC in the future by the directorate of construction in all governorates of KRG taking into consideration that the researchers used just 4 variables, it is possible that more variables might be used to detect issues and monitor the output of different cement factories in order to manage the quality of the companies' production.

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به کارهینانی وینهی کۆنترۆلی جۆری T^2 Hotelling فره گۆپاوی له کارگهی چیمهنتۆی بازیان

پوخته:

چیمهنتۆ پیکهاتهی سه رهکی کۆنکریت و زۆر بهی خانوو و تهلاری کوردی ده نوینیت به پشت به ستیت به پیکهاتهی کۆنکریتی. چیمهنتۆ تایبهتمهندی فیزیایی و کیمیایی ههیه که پشتیان پی ده به ستیت بۆ به دهستهینانی جۆریکی باش. وینهکانی کۆنترۆلی جۆری به کاردههینریت بۆ زانیی که پرۆسه که له ژیر کۆنترۆلدایه یان نا، ده کریت کاری له سهر بکریت له کارگهکانی چیمهنتۆ له حکومهتی ههریمی کوردستان. ئامانجی ئه م توژینهوه بریتیه له به کارهینانی وینهکانی کۆنترۆلی جۆری (QCC) یه که گۆپاوی و فره گۆپاوی بۆ چاودیریکردنی به رهمی چیمهنتۆ له کارگهی بازیان. شیکردنهوهی داتاگان له ریگهی Statgraphics v.16 به شیوهیهکی سه رهکی بۆ 4 تایبهتمهندی چیمهنتۆ (1 فیزیایی و 3

كيمياوى) به كارها تووه بۆ 100 پیدراو بۆ هەر تايبه تمه نديه ك. داتا كه دابه شكرا بۆ 25 گروپى لاوه كى و دواتر ويئنه $X\text{-bar}$ بۆ گۆراوه كان دۆزراوه ته وهو به راورديان پى كراوه له گه ل ئه نجامه كانى ويئنه كۆنترۆلى جۆرى فره يى به به كارهيئانى T^2 Hotelling. له ئه نجامدا ده ركه وت كه پرۆسه ي به ره مه يئانى چيمه نتۆ له زير كۆنترۆلدا نيبه به هۆى 2 تايبه تمه ندى كيميائى (SO_3 و Al_2O_3). راسپارده ده كريت ويئنه فره گۆراوى پيشنيازكراو له لايهن شوينى به رپرس به كاربه يئريت به تايبه تى ئه و به رپۆه به رايه تيانه ي كه په يوه ندييان به چاوديريكردنى كارگه چيمه نتۆيبه كان هه يه.

استخدام لوحات السيطرة النوعية T^2 Hotelling متعددة المتغيرات في معمل اسمنت بازيان

الملخص:

يمثل الأسمنت العنصر الرئيسي للمكون الخرساني ومعظم المنازل والمباني الكردية اعتمادا على الهيكل الخرساني. للأسمنت خصائص فيزيائية وكيميائية تعتمد عليها في الحصول على جودة جيدة. تستخدم لوحات السيطرة النوعية لمعرفة أن العملية تحت السيطرة أو لا، يمكن تطبيقها على معامل الأسمنت في حكومة إقليم كردستان (KRG). الهدف من هذا البحث هو استخدام لوحات السيطرة النوعية (QCC) أحادية المتغير ومتعددة المتغيرات لمراقبة إنتاج الأسمنت في مصنع بازيان. تم إجراء تحليل البيانات من خلال Statgraphics v.16 بصورة أساسية لأربعة خصائص أسمنتية (1 فيزيائية و 3 كيميائية) مع 100 مشاهدة لكل خاصية. تم تقسيم البيانات إلى 25 مجموعة فرعية ثم تم إيجاد لوحات $X\text{-bar}$ التي تم إنشاؤها للمتغيرات ومقارنتها بنتائج لوحات السيطرة النوعية المتعددة المتغيرات باستخدام T^2 Hotelling. تم التوصل إلى أن العملية ليست تحت السيطرة بسبب خاصيتين كيميائيتين (SO_3 و Al_2O_3). يوصى باستخدام لوحة متعدد المتغيرات المقترحة من قبل الجهات المسؤولة، وخاصة المديرية المعنية بمراقبة معامل الأسمنت.

Appendix: Table of subgroup averages of the data

Subgroup	compressive strength	Al ₂ O ₃	SO ₃	C ₃ S
1	36.68	6.16	3.30	20.57
2	38.58	6.53	3.27	20.82
3	36.98	6.93	3.23	20.33
4	41.18	6.08	3.35	21.20
5	42.10	6.08	3.32	20.38
6	40.83	5.91	3.26	20.24
7	40.18	6.30	3.42	21.19
8	41.45	6.42	3.35	21.30
9	38.33	6.75	3.30	21.14
10	41.45	5.92	3.24	20.47
11	39.35	6.69	3.20	20.57
12	40.93	6.69	3.27	20.65
13	38.70	6.79	3.31	21.04
14	43.10	6.72	3.29	21.25
15	46.28	6.69	3.43	15.89
16	45.30	5.32	3.52	15.72
17	39.13	6.96	3.33	20.84
18	41.45	5.11	3.45	21.06
19	44.68	* 3.85	3.64	21.53
20	42.58	4.44	3.92	21.04
21	46.35	4.82	3.73	20.84
22	44.53	* 3.17	* 4.55662	20.80
23	44.40	* 3.83	* 4.0675	15.72
24	44.53	4.68	* 4.00	* 15.56
25	42.23	4.31	3.68	15.82

* The averages that are out of control.