

## Vibration Analysis of Composite Laminated Plates Using Analytical and APDL ANSYS Codes

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#### ABSTRACT

The aim of this work is to investigate analytical (Classical theory) and [ANSYS Parametric Design Language (APDL)] analysis of laminate composite plate. The free vibration analysis with two boundary conditions simply supported (SSSS) & clamped ends (CCCC) are taken into account in this work. The equations of motion that governs the predication of dynamic behavior of cross and angle ply composite are solved theoretically using [Matlab] software. Three degrees of freedom are included in computing the natural frequency and mode shapes of cross and angle ply laminate. The significant effect of various parameters such as number of layers of fibers, aspect ratios, fiber orientation, and modulus ratio on natural frequency is evaluated theoretically and APDL (ANSYS). The numerical analysis is carried out by commercially available ANSYS software. The boundary conditions can significantly affect the natural frequency of the composite laminate plate due to the restraint effect at the edges. In this work a relatively new modeling technique will be adopted. Good agreement is achieved between the analytical and ANSYS results.

## **1. Introduction**

Combining one or more materials are the advanced class of formed materials that called laminated composite, having different physical and chemical properties. The property of the material for each layer remains constant with different fiber angles for tailor ability. Demand for the development of mechanical structures with weight to high strength ratios, stiffness, resistance to corrosion, long fatigue life and stealth characteristics has increased recently among industrial companies which leads to build lightweight aerospace structures, tall buildings and long bridges. The objectives of the construction of mechanical structures with high specific strengths are to optimize loading capacity, energy consumption and material costs. Nonetheless, analysis of free vibration study of composite laminated plates is very important for inspection damage and modifying the resonant frequency, for structures that associated with vibration problems. With frequently increasing the use of composites, especially in the sector of automobile and aerospace, vibration problems study has come to light in laminated plates that become important. The dynamic behavior prediction of laminated composite plates plays an important role in the structural composite applications. (Itishree Mishra & Shishir Kumar Sahu ,2012) presented components dynamic behavior that made from driven woven fiber composite plates materials industry. Different geometrical parameters effects including aspect ratio, layers number and fibers angle of driven woven fiber industry composite plates in free-free boundary condition in details are studied. (Itishree Mishra\* and Shishir Kumar Sahu,2015) computed modal testing using FFT analyzer of woven fiber glass/epoxy laminated composite plates. Different geometrical parameters effects including fiber orientation angles, number of layers, aspect ratio and deferent (BCs) boundary conditions of industry driven woven fiber composite plates are studied in details. (Pushparaj Pingulkar and Suresha B.,2016) studied fiber volume fraction effect on the frequencies of the modal of PMC plates using software “ANSYS” the commercial finite element. The using materials were fiberglass S2, E-glass, T300 carbon and unidirectional T800 carbon fibers reinforced with matrix vinyl ester epoxy. The simulated plates for quasi-isotropic stacking sequence consisting layers having fiber angle orientations of  $0^\circ$ ,  $+45^\circ$ ,  $-45^\circ$ , and  $90^\circ$ . Hybridization effect was also studied by layers stacking of carbon fibers and glass fiber one on top of the other. In the end the effect on the fiber angle of the outermost layers on the natural frequencies was investigated.

(A.K. Nayaka, S.S.J. Moyb, R.A. Shenoi,2002) deals with the Reddys new type elements family based on the theory of Reddy’s higher-order that used for sandwich plates



composite in the free vibration analysis. The effect of boundary conditions, plate geometry, material property and lamination scheme on the natural frequencies of composite sandwich plates is investigated. For better understanding the dynamic behavior of composite sandwich plates such a study is necessary. (E. V. Prasad and S. K. Sahu,2018) focused on the elastic properties determination from numerical and experimental vibration analysis of woven Glass FMLs (GFMLs) tensile testing. Various parameters effects on the natural frequency of the FML [Fiber Metal Laminate (FML)] plates using aluminum and woven glass fiber. (Devidas R. Patil 1, P.G.Damle 2, Dr. D.S.DeshmukhIn,2014) computed problem modeled in ANSYS platform with Finite Element formulation and confirmation on the terms related to the mass matrices and stiffness. Then, a variety of results are offered to show the suitability of the present problem to different types of plates under free vibration conditions. (Pushpendra k. kushwaha1 and jyoti vimal,2014) analyzed the laminated composite plates vibrational analysis by using method of Finite Element and results from numerical have been computed for the effect of thickness ratio, number of layers of plate, various boundary conditions, various aspect ratio and different angle of fibre orientation of laminated composite plate. (Sharayu U. Ratnaparkhi, 1 S.S. Sarnobat2 ,2013) discussed analysis of vibration for Glass/Epoxy composite plate under condition of Free boundary, the effect of factors such as layers number, fiber angle orientation and aspect ratio on the natural frequency were carried out. (Venkatachalam Gopalan and Vimalanand Suthenthiraveerappa,2021) performed experimental investigation with numerical to perform the free vibration analysis of (WFBE) woven Flax/Bio epoxy composite plates. To carry out the analysis of vibration using the numerical approach, the constants of elastic were resolute by regular mechanical tests and the excitation of impulse for vibration-based approach. (Dr.Wedad Majed Ibrahim Rafat Assad Ghani,2017) used Rayleigh–Ritz technique for rectangular isotropic , symmetric and anti-symmetric laminated cross and angle ply composite plate with general elastic supports along its edges and calculate the natural frequencies for them. (Philippe Vidal, Laurent Gallimard, Olivier Polit,2019) dealt with the free vibration analysis of laminated composite plates through a variable separation approach. The domains displacement is approximated as a sum of separated functions of the in-plane transverse coordinate  $z$  with coordinates  $x$  and  $y$ . (Khaldoon F. Brethee,2009) studied

the analysis of free vibration laminated composite plates which has its significant effect for damage inspection and modifying the frequency of resonant, symmetric and anti-symmetric laminated composite plates free vibration analysis produced using ANSYS computer program. Different effect parameters such as number of layers, thickness ratio, fiber angle orientation and boundary condition has been considered on the laminated plate.

## 2. Analytical Solutions of Rectangular Laminated Plates

### 2.1 Governing Equations

In this investigation, analytical solutions of antisymmetric cross-ply and angle-ply laminated plates based on the Classical Laminated Plate Theory (CLPT) are analyzed. The Navier method is used depending on the boundary conditions. In this case the von Karman nonlinear terms in the strain-displacement relations are omitted. Before beginning with the derivation of the exact solutions, it is useful to express the governing equations in terms of the generalized displacements of the theory. The linear equations of motion of the (CLPT) (J.N. REDDY, 2004)

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} + 2 A_{16} \frac{\partial^2 u_0}{\partial x \partial y} + A_{66} \frac{\partial^2 u_0}{\partial y^2} + A_{16} \frac{\partial^2 v_0}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 v_0}{\partial x \partial y} + A_{26} \frac{\partial^2 v_0}{\partial y^2} - \left[ B_{11} \frac{\partial^3 \omega_0}{\partial x^3} + 3 B_{16} \frac{\partial^3 \omega_0}{\partial x^2 \partial y} + (B_{12} + 2 B_{66}) \frac{\partial^3 \omega_0}{\partial x \partial y^2} + B_{26} \frac{\partial^3 \omega_0}{\partial y^3} \right] = I_0 \ddot{u}_0 - I_1 \frac{\partial \dot{\omega}_0}{\partial x} \quad (1)$$

$$A_{16} \frac{\partial^2 u_0}{\partial x^2} + (A_{12} + A_{66}) \frac{\partial^2 u_0}{\partial x \partial y} + A_{26} \frac{\partial^2 u_0}{\partial y^2} + A_{66} \frac{\partial^2 v_0}{\partial x^2} + 2 A_{26} \frac{\partial^2 v_0}{\partial x \partial y} + A_{22} \frac{\partial^2 v_0}{\partial y^2} - \left[ B_{16} \frac{\partial^3 \omega_0}{\partial x^3} + (B_{12} + 2 B_{66}) \frac{\partial^3 \omega_0}{\partial x^2 \partial y} + 3 B_{26} \frac{\partial^3 \omega_0}{\partial x \partial y^2} + B_{22} \frac{\partial^3 \omega_0}{\partial y^3} \right] = I_0 \ddot{v}_0 - I_1 \frac{\partial \dot{\omega}_0}{\partial y} \quad (2)$$

$$B_{11} \frac{\partial^3 u_0}{\partial x^3} + 3 B_{16} \frac{\partial^3 u_0}{\partial x^2 \partial y} + (B_{12} + 2 B_{66}) \frac{\partial^3 u_0}{\partial x \partial y^2} + B_{26} \frac{\partial^3 u_0}{\partial y^3} + B_{16} \frac{\partial^3 v_0}{\partial x^3} + (B_{12} + 2 B_{66}) \frac{\partial^3 v_0}{\partial x^2 \partial y} + 3 B_{26} \frac{\partial^3 v_0}{\partial x \partial y^2} + B_{22} \frac{\partial^3 v_0}{\partial y^3} - \left[ D_{11} \frac{\partial^4 \omega_0}{\partial x^4} + 4 D_{16} \frac{\partial^4 \omega_0}{\partial x^3 \partial y} + 2 (D_{12} + 2 D_{66}) \frac{\partial^4 \omega_0}{\partial x^2 \partial y^2} + 4 D_{26} \frac{\partial^4 \omega_0}{\partial x \partial y^3} + D_{22} \frac{\partial^4 \omega_0}{\partial y^4} \right] + \hat{N}_{xx} \frac{\partial^2 \omega_0}{\partial x^2} + 2 \hat{N}_{xy} \frac{\partial^2 \omega_0}{\partial x \partial y} + \hat{N}_{yy} \frac{\partial^2 \omega_0}{\partial y^2} + q = I_1 \left( \frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) + I_0 \ddot{\omega}_0 - I_2 \left( \frac{\partial^2 \dot{\omega}_0}{\partial x^2} + \frac{\partial^2 \dot{\omega}_0}{\partial y^2} \right) \quad (3)$$

Where  $\hat{N}_{xx}, \hat{N}_{xy}, \hat{N}_{yy}$  denote the applied edge forces (see Figure 1)

Equations (1-3) can be cast in differential operator form as

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ \omega_0 \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \\ \ddot{\omega}_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q \end{Bmatrix} \quad (4)$$

Where coefficients  $c_{ij}$  are defined by

$$\begin{aligned} c_{11} &= A_{11}d_x^2 + 2A_{16}d_xd_y + A_{66}d_y^2 \\ c_{12} &= A_{16}d_x^2 + (A_{12} + A_{66})d_xd_y + A_{26}d_y^2 \\ c_{13} &= -[B_{11}d_x^3 + 3B_{16}d_x^2d_y + (B_{12} + 2B_{66})d_xd_y^2 + B_{26}d_y^3] \\ c_{22} &= A_{66}d_x^2 + 2A_{26}d_xd_y + A_{22}d_y^2 \\ c_{23} &= -[B_{16}d_x^3 + (B_{12} + 2B_{66})d_x^2d_y + 3B_{26}d_xd_y^2 + B_{22}d_y^3] \\ c_{33} &= D_{11}d_x^4 + 4D_{16}d_x^3d_y + 2(D_{12} + 2D_{66})d_x^2d_y^2 + 4D_{26}d_xd_y^3 + D_{22}d_y^4 \\ &- [\hat{N}_{xx}d_x^2 + 2\hat{N}_{xy}d_xd_y + \hat{N}_{yy}d_y^2] \end{aligned} \quad (5)$$

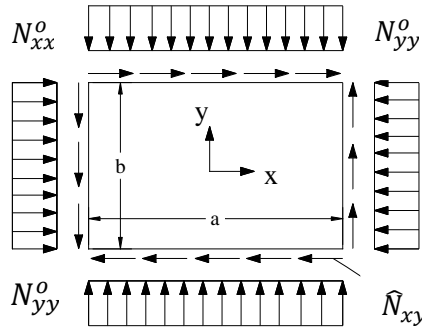


Figure (1): A plate with applied edge forces ( $\hat{N}_{xx} = -N_{xx}^o$ ,  $\hat{N}_{yy} = -N_{yy}^o$ ).

Coefficients  $m_{ij}$  is defined by

$$\begin{aligned} m_{11} &= -I_0 d_t^2, \quad m_{13} = I_1 d_x d_t^2, \quad m_{22} = -I_0 d_t^2 \\ m_{23} &= I_1 d_y d_t^2, \quad m_{33} = I_0 d_t^2 - I_2 d_t^2 (d_x^2 + d_y^2) \end{aligned} \quad (6)$$

and  $d_t^i$  denote the differential operators

$$d_x^i = \frac{\partial^i}{\partial x^i}, \quad d_y^i = \frac{\partial^i}{\partial y^i}, \quad d_t^i = \frac{\partial^i}{\partial t^i}, \quad (i = 1, 2, 3, 4) \quad (7)$$

Where  $A_{ij}$  are called extensional stiffnesses,  $D_{ij}$  the bending stiffnesses, and  $B_{ij}$  the bending-extensional stiffnesses, which are defined in terms of the lamina stiffnesses  $\bar{Q}_{ij}^{(k)}$  as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}(1, z, z^2) dz = \sum_{K=1}^N \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)}(1, z, z^2) dz \quad (8)$$

$$A_{ij} = \sum_{K=1}^N \bar{Q}_{ij}^{(k)} (Z_{k+1} - Z_k), \quad B_{ij} = \frac{1}{2} \sum_{K=1}^N \bar{Q}_{ij}^{(k)} (Z_{k+1}^2 - Z_k^2)$$

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned} \quad (9)$$

### 3. Navier Solutions of Antisymmetric Cross-Ply Laminates

#### 3.1 Boundary Conditions

In the Navier method the generalized displacements are expanded in a double trigonometric series in terms of unknown parameters. The choice of the functions in the series is restricted to those which satisfy the boundary conditions of the problem. Substitution of the displacement expansions into the governing equations should result in a unique, invertible, set of algebraic equations among the parameters of the expansion. Otherwise, the Navier solution cannot be developed for the problem.

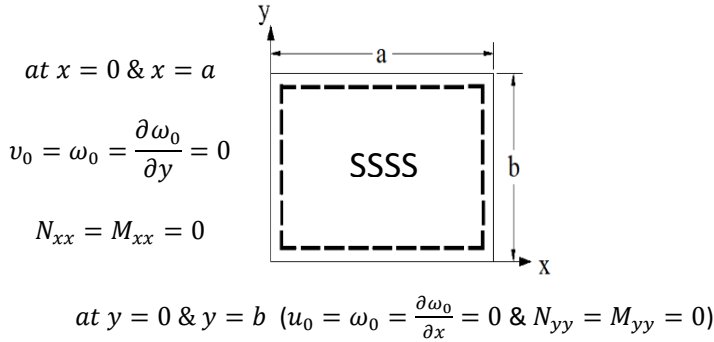


Figure (2): Simply supported boundary conditions, SSSS, used in the analytical solutions of rectangular laminated plates

The Navier solutions can be developed for rectangular laminates with one set of simply supported boundary conditions. The geometry, laminate coordinate system, and simply supported boundary conditions are given in Figure 2. In equations (10), a and b denote the in-plane dimensions along the x and y directions of a rectangular laminate. The origin of the coordinate system is taken at the lower left corner of the midplane, as shown in Figure 2. As will be shown in the following sections, the Navier solutions using SSSS boundary conditions can be obtained only for laminates whose stiffnesses  $A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26},$  and  $A_{45}$  are zero because the layer stiffnesses  $\bar{Q}_{16}, \bar{Q}_{26}, \bar{Q}_{45}$  are zero for cross-ply angles (0/90). Similarly, the Navier solutions using SSSS boundary conditions can be obtained for angle-ply laminates whose stiffnesses  $A_{16}, A_{26}, B_{11}, B_{12}, B_{22}, B_{66}, D_{16}, D_{26}$  and  $A_{45}$  are zero. For a fixed laminate thickness, the stiffnesses  $B_{16}$  and  $B_{26}$  go to zero as the number of layers in the laminate increases. The boundary conditions in Eq. (10) imply the following SSSS boundary conditions on the displacements and stress resultants of the classical laminate theory (J.N. REDDY, 2004).

$$\begin{aligned}
 u_0(x, 0, t) = 0, \quad u_0(x, b, t) = 0, \quad v_0(0, y, t) = 0, \quad v_0(a, y, t) = 0 \\
 \omega_0(x, 0, t) = 0, \quad \omega_0(x, b, t) = 0, \quad \omega_0(0, y, t) = 0, \quad \omega_0(a, y, t) = 0 \\
 \left. \frac{\partial \omega_0}{\partial x} \right|_{(x,0,t)} = 0, \quad \left. \frac{\partial \omega_0}{\partial x} \right|_{(x,b,t)} = 0, \quad \left. \frac{\partial \omega_0}{\partial y} \right|_{(0,y,t)} = 0, \quad \left. \frac{\partial \omega_0}{\partial y} \right|_{(a,y,t)} = 0
 \end{aligned} \tag{10}$$

The displacement boundary conditions of SSSS in Eq. (10) are satisfied by assuming the following form of the displacements

$$u_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{mn}(t) \cos \alpha x \sin \beta y \tag{11}$$

$$v_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} V_{mn}(t) \sin \alpha x \cos \beta y \tag{12}$$

$$\omega_0(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn}(t) \sin \alpha x \sin \beta y \tag{13}$$

Where  $\alpha = m\pi/a$  and  $\beta = n\pi/b$  and  $(U_{mn}, V_{mn}, W_{mn})$  are coefficients to be determined.

#### 4. Equation of Motion

The solution of governing equations (1-3) results as follows:

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [(-A_{11}\alpha^2 + A_{66}\beta^2)U_{mn} - (A_{12} + A_{66})\alpha\beta V_{mn} + \{B_{11}\alpha^3(B_{12} + 2B_{66})\alpha\beta^2\}W_{mn} - I_0\ddot{U}_{mn} + I_1\alpha\ddot{W}_{mn}] \cos \alpha x \sin \beta y = 0 \tag{14}$$

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [-(A_{12} + A_{66})\alpha\beta U_{mn} - (A_{66}\alpha^2 + A_{22}\beta^2)V_{mn} + \{(B_{12} + 2B_{66})\alpha^2\beta + B_{22}\beta^3\}W_{mn} - I_0\ddot{V}_{mn} + I_1\ddot{W}_{mn}\beta] \sin \alpha x \cos \beta y = 0 \tag{15}$$

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [(B_{11}\alpha^3 + \tilde{B}_{12}\alpha\beta^2)U_{mn}(t) + (\tilde{B}_{12}\alpha^2\beta + B_{22}\beta^3)V_{mn} - (D_{11}\alpha^4 + 2\tilde{D}_{12}\alpha^2\beta^2 + D_{22}\beta^4)W_{mn} - (\alpha^2\tilde{N}_{xx} + \beta^2\tilde{N}_{yy})W_{mn} + I_1\alpha\ddot{U}_{mn} + I_1\beta\ddot{V}_{mn} - (I_0 + I_2(\alpha^2 + \beta^2))\ddot{W}_{mn}] \sin \alpha x \sin \beta y = -q(x, y) \tag{16}$$

Where  $q(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{mn}(t) \sin \alpha x \sin \beta y$

Note that the edge shear force  $\tilde{N}_{xy}$  is necessarily zero. In addition, inertia  $I_1$  is zero.

Equations (14, 15, & 16) can be expressed in matrix form as

$$\begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} + \hat{s}_{33} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} + \begin{bmatrix} \hat{m}_{11} & 0 & -I_1\alpha \\ 0 & \hat{m}_{22} & -I_1\beta \\ -I_1\alpha & -I_1\beta & \hat{m}_{33} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \end{Bmatrix} \tag{17}$$

$$\begin{aligned} \hat{c}_{11} &= (A_{11}\alpha^2 + A_{66}\beta^2) \\ \hat{c}_{12} &= (A_{12} + A_{66})\alpha\beta \\ \hat{c}_{13} &= -B_{11}\alpha^3 - (B_{12} + 2B_{66})\alpha\beta^2 \\ \hat{c}_{22} &= (A_{66}\alpha^2 + A_{22}\beta^2) \\ \hat{c}_{23} &= -(B_{12} + 2B_{66})\alpha^2\beta - B_{22}\beta^3 \\ \hat{c}_{33} &= D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \end{aligned} \tag{18}$$



$$\tilde{s}_{33} = \alpha^2 \tilde{N}_{xx} + \beta^2 \tilde{N}_{yy} \tag{19}$$

$$\hat{m}_{11} = \hat{m}_{22} = I_0 \quad \hat{m}_{33} = (I_0 + I_2(\alpha^2 + \beta^2)) \quad \text{where } \alpha = m\pi/a \quad \text{and } \beta = n\pi/b \tag{20}$$

### 5. Navier Solutions of Antisymmetric Angle-Ply Laminates

Similarly, the derivation of governing equations for angle-ply laminates gives the following matrix equation

$$\begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} + \hat{s}_{33} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} + \begin{bmatrix} \hat{m}_{11} & 0 & 0 \\ 0 & \hat{m}_{22} & 0 \\ 0 & 0 & \hat{m}_{33} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \end{Bmatrix} \tag{21}$$

$$\begin{aligned} \hat{c}_{11} &= (A_{11} \alpha^2 + A_{66} \beta^2) \\ \hat{c}_{12} &= (A_{12} + A_{66})\alpha\beta \\ \hat{c}_{13} &= -(3 B_{16} \alpha^2 + B_{26} \beta^2) \beta \\ \hat{c}_{22} &= (A_{66} \alpha^2 + A_{22} \beta^2) \\ \hat{c}_{23} &= -(B_{16} \alpha^2 + 3 B_{26} \beta^2) \alpha \\ \hat{c}_{33} &= D_{11} \alpha^4 + 2 (D_{12} + 2 D_{66}) \alpha^2 \beta^2 + D_{22} \beta^4 \\ \tilde{s}_{33} &= \alpha^2 \tilde{N}_{xx} + \beta^2 \tilde{N}_{yy} \\ \hat{m}_{11} &= \hat{m}_{22} = I_0 \\ \hat{m}_{33} &= (I_0 + I_2 (\alpha^2 + \beta^2)) \end{aligned} \tag{22}$$

### 6. Vibration Solution

For free vibration Eq. (21) reduces to the eigenvalue problem

$$\left( \begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} \hat{m}_{11} & 0 & 0 \\ 0 & \hat{m}_{22} & 0 \\ 0 & 0 & \hat{m}_{33} \end{bmatrix} \right) \begin{Bmatrix} U_{mn}^0 \\ V_{mn}^0 \\ W_{mn}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

For a nontrivial solution,  $U_{mn}^0 \neq 0$ ,  $V_{mn}^0 \neq 0$ ,  $W_{mn}^0 \neq 0$ , the determinant of the coefficient matrix in (24) should be zero, which yields the characteristic polynomial

$$-p\lambda^3 + q\lambda^2 - r\lambda + s = 0 \tag{24}$$

$\hat{c}_{ij}$  &  $\hat{m}_{ij}$  are defined in Equations (22). Setting the determinant of the coefficient matrix in (24) to zero, we obtain the cubic characteristics polynomial

$$\begin{aligned}
 p &= \begin{vmatrix} \hat{m}_{11} & 0 & 0 \\ 0 & \hat{m}_{22} & 0 \\ 0 & 0 & \hat{m}_{33} \end{vmatrix}, & s &= \begin{vmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} \end{vmatrix} \\
 q &= \begin{vmatrix} \hat{c}_{11} & 0 & 0 \\ \hat{c}_{12} & \hat{m}_{22} & 0 \\ \hat{c}_{13} & 0 & \hat{m}_{33} \end{vmatrix} + \begin{vmatrix} \hat{m}_{11} & \hat{c}_{12} & 0 \\ 0 & \hat{c}_{22} & 0 \\ 0 & \hat{c}_{23} & \hat{m}_{33} \end{vmatrix} + \begin{vmatrix} \hat{m}_{11} & 0 & \hat{c}_{13} \\ 0 & \hat{m}_{22} & \hat{c}_{23} \\ 0 & 0 & \hat{c}_{33} \end{vmatrix} \\
 r &= \begin{vmatrix} \hat{c}_{11} & \hat{c}_{12} & 0 \\ \hat{c}_{12} & \hat{c}_{22} & 0 \\ \hat{c}_{13} & \hat{c}_{23} & \hat{m}_{33} \end{vmatrix} + \begin{vmatrix} \hat{c}_{11} & 0 & \hat{c}_{13} \\ \hat{c}_{12} & \hat{m}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & 0 & \hat{c}_{33} \end{vmatrix} + \begin{vmatrix} \hat{m}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ 0 & \hat{c}_{22} & \hat{c}_{23} \\ 0 & \hat{c}_{23} & \hat{c}_{33} \end{vmatrix} \tag{25}
 \end{aligned}$$

Where eigenvalue  $\lambda = \omega^2$ ,

If the in-plane inertias are neglected (*i. e.*,  $\hat{m}_{11} = \hat{m}_{22} = 0$ ), Eq. (23) yields

$$\omega^2 = \frac{1}{\hat{m}_{33}} \left( \hat{c}_{33} - \frac{\hat{c}_{13}\hat{c}_{22} - \hat{c}_{23}\hat{c}_{12}}{\hat{c}_{11}\hat{c}_{22} - \hat{c}_{12}\hat{c}_{21}} \hat{c}_{13} - \frac{\hat{c}_{11}\hat{c}_{23} - \hat{c}_{12}\hat{c}_{13}}{\hat{c}_{11}\hat{c}_{22} - \hat{c}_{12}\hat{c}_{21}} \hat{c}_{23} \right) \tag{26}$$

Note that  $\omega$  is a function of the mode numbers (m,n) because the coefficients  $\hat{c}_{ij}$  depend on (m,n) as shown in equations (21&22). The smallest of the frequencies is called the fundamental frequency. The smallest frequency might occur for values  $m = n = 1$ . Nondimensionalized frequencies,  $\bar{\omega}_{mn} = \omega_{mn} \left( \frac{b^2}{\pi^2} \right) \sqrt{\rho h / D_{22}}$ , of specially orthotropic and antisymmetric cross-ply square laminates are presented in table 1 for modulus ratios ( $E_1/E_2 = 10, 20, 30, \text{ and } 40$ ) ( $G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \nu_{12} = 0.25$ ). All layers are of equal thickness. Results are presented for  $m, n = 1, 2, 3$ . The fundamental frequency increases with modular ratio. Also, the natural frequency in Hz is computed using the relation  $f_n = \frac{\omega_{mn}}{2\pi}$ . Analytical program is carried out in many different steps as shown in Figure 4 to solve equation 23 using eigen value problem to evaluate the eigen values (natural frequencies) and eigen vectors (mode shapes) of rectangular composite plate.

## 7. Finite Element Modeling

Solid structural 3D layered element is used to model the composite laminate plate into many elements as shown in Figure (3). ANSYS software version 19 is used to model composite laminated structure using finite element technique.

### 7.1 Material Properties

The first step in the Finite Element Analysis is to simulate the dynamic characteristics of the composite structures using ANSYS macro codes. The model geometry properties have described using these macro codes.

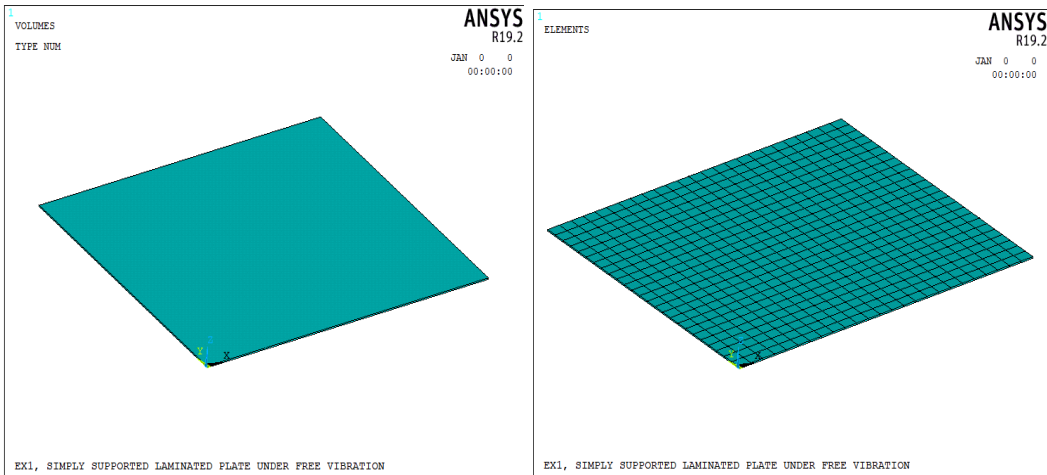


Figure (3): Configuration for the laminate structure of the composite plate

The Flowchart Matlab Program which describes the steps of analyzing the analytical differential equations of composite laminate plates is shown in Figure (4).

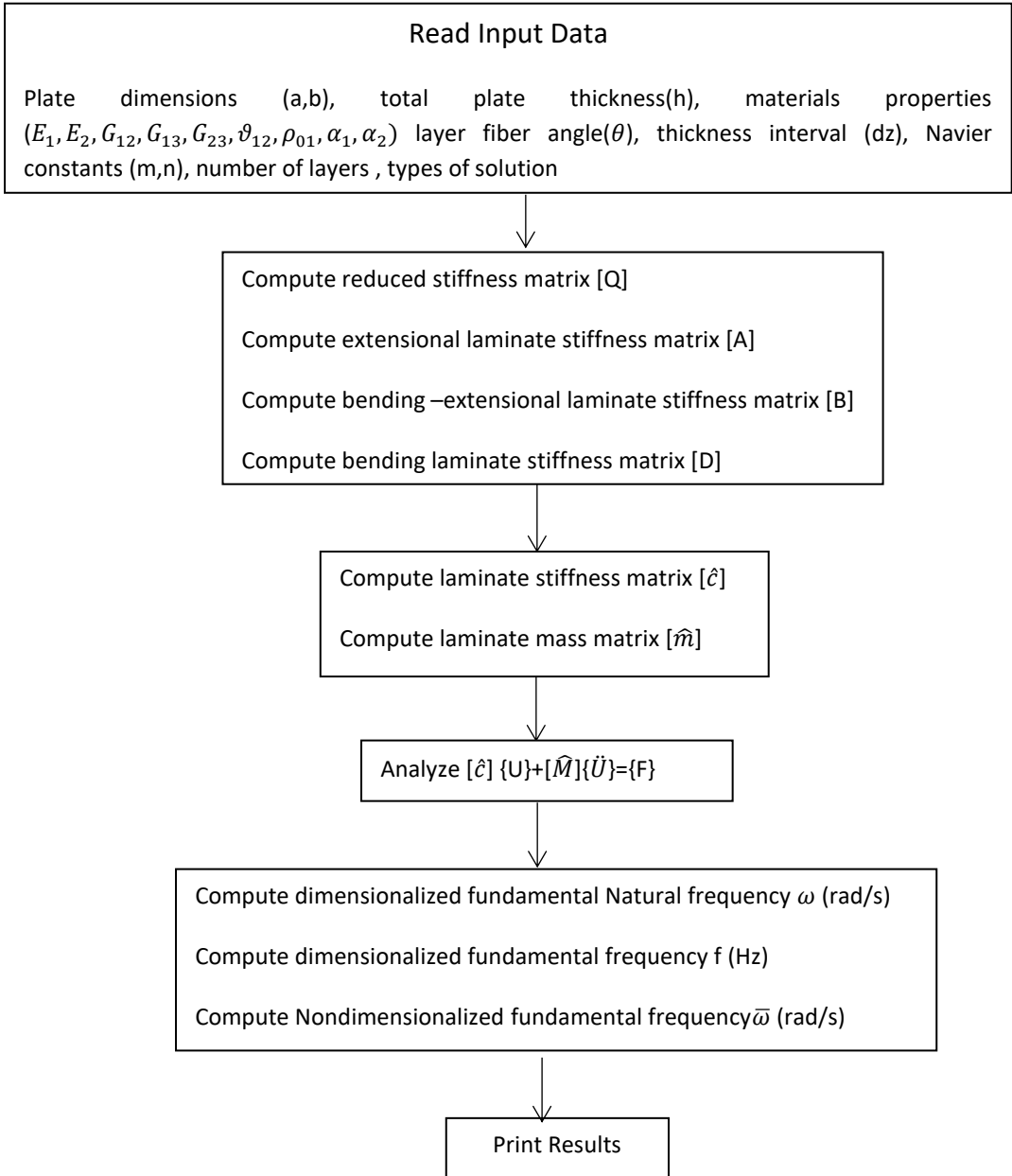


Figure (4): Flowchart of Analytical Matlab Program

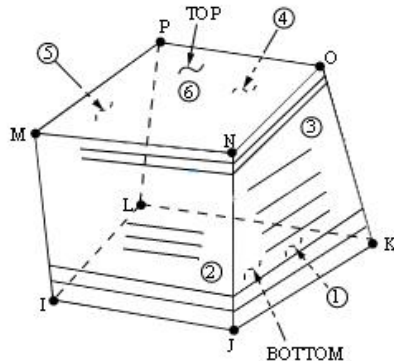


Figure (5): Element type: SOLID185 layered structural

## 7.2 Analysis Setup

The macro ANSYS codes present many different possibilities such as creates finite element divisions and conduct analysis of the elements to get an appropriate solution of structure model. The structural model in this work is created using ANSYS Parametric Design Language (APDL) macro code. The steps of creating a finite element model for composite laminated are described in the following sub-sections.

- 1- The initiation of simulation environment.
- 2- The definitions of parameters.
- 3- The modeling of solid geometry.
- 4- The meshing of solid geometry.
- 5- Applying boundary and initial conditions.

The most important steps of the project coding by APDL ANSYS in the following.

## 7.3 Initializing

Initializing of modeling and simulation process starts exiting normally from the processor and clearing the ANSYS database. The title of the job is stated, while successive commands suppress extended interpreted data input and place comments in the output. To begin the modeling phase, the pre-processor is engaged. To prevent ANSYS from displaying irrelevant warning messages, shape checking is carried out in silent mode. The shape checking limit is also increased from the default value.

**7.4 Composite Laminate Plate Dimensions and Material**

Length, thickness and width of the support plate are declared with precision. Also, the material properties reinforced polymer are stated, including Young's modulus and density, Poisson's ratio, as shown in Table (1). We used a composite plate of size (25 mm x 25 mm) and constant thicknesses (1 mm).

**7.5 Assigning Element Type**

SOLID185 and SOLID186 elements are used as layered structural elements for composite laminated plates as shown in Figure (5) (KAMEL, M.A., IBRAHIM, K. AND AHMED, A.E.M. 2019). Additional degree of freedom is controlled by the element type command ET. The defined element type ET of composite material is given in Table (2) as APDL macro codes.

Table (1): Material properties of composite laminate plate (J.N. REDDY, 2004).

Material Properties	
Young's modulus $E_1$	$10E_2$
Young's modulus $E_2$	$2.1 * 10^6 \text{N/cm}^2$
Young's modulus $E_3$	$2.1 * 10^6 \text{N/cm}^2$
Shear modulus $G_{12}$	$0.5E_2$
Poisson's ratio $\nu_{12}$	0.25
Poisson's ratio $\nu_{13}$	0.25
Density $\rho$	$8 * 10^{-6} \text{ N. sec}^2/\text{cm}^4$

Table (2): Element definition codes

<b>/PREP7</b>	! start the preprocessing
<b>ET,1,185</b>	! chooses SOLID185 element for analysis composite material
<b>KEYOPT,1,3,1</b>	! Key option #3 = 1 LAYERED SOLID ELEMENTS
<b>KEYOPT,1,2,2</b>	! Key option #2 = 2 ENHANCED STRAIN FORMULATION
<b>KEYOPT,1,8,1</b>	! Key option #8 = 1 STORE DATA FOR ALL LAYERS

### 7.6 Plate Model

Several steps are necessary to create a plate model. Firstly, composite laminated block is created with proper dimensions as given in Table (3) using APDL macro codes. According to both number of the layers and type of element, the plate is converted to sub-sections elements. This is performed using meshing command. The model structure is divided into many elements as shown in Figure (3). The composite material properties used in this work are given in Table (1). The material characteristics of composite laminate are given in Table (4) as APDL macro codes. The command MP inputs material properties like density and Poisson's ratio, at constant stress or Young's modulus if applicable. Table (5) gives layers thickness, layers numbers, and angles of fiber as APDL macro codes for the composite laminated plate.

Table (3): Key points generation

<b>/PREP7</b>	! start the preprocessing
<b>K,1,0,0</b>	! corner key points of full (volume)
<b>K,2,a,0</b>	! second corner key points of full (volume)
<b>K,3,a,b</b>	! third corner key points of full (volume)
<b>K,4,0, b</b>	! fourth corner key points of full (volume)
<b>KGEN,2,1,4,1,,,0.1</b>	! Key points generation
<b>L,1,5</b>	! connect point 1 with point 5
<b>*REPEAT,4,1,1</b>	! repeat generating points
<b>LESIZE,ALL,,,1</b>	
<b>V,1,2,3,4,5,6,7,8</b>	! generate volume model

### 7.7 Meshing Solid Model

The composite materials and elements are assigned to geometric shape structure to create a meshed finite element model. ESIZE command is used to define the finite element size geometry. ALLSEL command is used to choose all shapes in model and VPLOT command is utilized to re-plot the model to the screen.

Command VMESH initiates the meshing of all volumes as given in Table (5). Finally, all nodes are selected after meshing. All characteristic and dimensions of composite laminate are selected before assigning the element type to the composite materials. The layered element type used in this work is SOLID185 element. The frequency and

mode shapes of composite laminate are evaluated numerically using modal analysis technique for different layers. Figure (3) presents a discretized finite elements model of laminate plate.

Table (4): Material Properties codes

<b>/PREP7</b>	! start the preprocessing
<b>MP, EX,1,21E6</b>	! Define Material 1 #: $E_1=10E_2$
<b>MP, EY,1,2.1E6</b>	! Define Material 1#: $E_2=2.1 * 10^6 \text{N/cm}^2$
<b>MP, EZ,1,2.1E6</b>	! $E_3=E_2$ ASSUMED
<b>MP, GXY,1,1.05E6</b>	! Define Material 1#: shear modulus xy-plane $G_{12} = 0.5E_2$
<b>MP, GYZ,1,0.42E6</b>	! Define Material 1#: shear modulus xz-plane $G_{23} = 0.2E_2$
<b>MP, GXZ,1,1.05E6</b>	! Define Material 1#: shear modulus xy-plane $G_{13} = 0.5E_2$
<b>MP, PRXY,1,0.25</b>	! Define Material 1#: Poisson coefficient $\nu_1 = 0.25$
<b>MP, PRYZ,1,0.01</b>	! Define Material 1#: Poisson coefficient $\nu_2 = 0.25$
<b>MP, PRXZ,1,0.25</b>	! Define Material 1#: Poisson coefficient $\nu_3 = 0.25$
<b>MP, DENS,1,8E-6</b>	! Define Material 1#: Density for composite material

Table (5): Orientation angles', Thickness, and meshing Composite layers codes

<b>/PREP7</b>	! start the preprocessing
<b>SECTYPE,1, SHELL</b>	! set #1, [0/90/0]
<b>SECDATA,0.53e-3,1,0</b>	! 1st LAYER 1: Thk=0.53e-3m, THETA = 0°
<b>SECDATA,0.53e-3,1,90</b>	! 2nd LAYER 2: Thk=0.53e-3m, THETA = 90°
<b>SECDATA,0.53e-3,1,0</b>	! 3rd LAYER 4: Thk=0.53e-3m, THETA = 0°
<b>ESIZE, ,25</b>	! 25X25 MESH USING FULL MODEL
<b>VMESH,1</b>	! initiates the meshing of all volumes

### 7.8 Defining Boundary Conditions

Various code segments are used to apply boundary conditions on the model edges. Simply supported boundary conditions (S) are used to support four edges of the laminated plate as shown in Figure (2). Also, clamped boundary condition (C) is applied to edges of fixed ends. Four ends of laminate are clamped or supported as simply boundary conditions. NSEL command is used to select nodes in the laminated geometry. D command is used as APDL macro code to assign zero displacement degree of freedom constraints to the laminate edges. After selecting all constrained nodes, the processor stage is left by the FINISH command, and the solution step



entered afterwards. Table (6) gives macro ANSYS codes of fixed and simply supported boundary conditions for coordinates (x=0, a) and (y=0, b).

**Table (6): Boundary conditions of composite plate codes**

<b>/PREP7</b>	! simply supported boundary conditions (SSSS)
<b>/PREP7</b>	! start the preprocessing
<b>NSEL, S, LOC, X,0</b>	! select nodes at edge (X=0)
<b>D, ALL, UZ,,,,,UY</b>	! apply deflections (UY=UZ=0) at (X=0)
<b>NSEL, S, LOC, X, a</b>	! select nodes at edge (X=a)
<b>D, ALL, UZ,,,,,UY</b>	! apply deflections (UY=UZ=0) at (X=a)
<b>NSEL, S, LOC, Y,0</b>	! select nodes at edge (Y=0)
<b>D, ALL, UX,,,,,UX</b>	! apply deflections (UX=UZ=0) at (Y=0)
<b>NSEL, S, LOC, Y, b</b>	! select nodes at edge (Y=b)
<b>D, ALL, UX,,,,,UX</b>	! apply deflections (UX=UZ=0) at (Y=b)
<b>! clamped boundary conditions (CCCC)</b>	
<b>NSEL, S, LOC, X,0</b>	! select nodes at edge (X=0)
<b>D, ALL, ALL</b>	! apply deflections (UX=UY=UZ=0) at (X=0)
<b>NSEL, S, LOC, X, a</b>	! select nodes at edge (X=a)
<b>D, ALL, ALL</b>	! apply deflections (UX=UY=UZ=0) at (X=a)
<b>NSEL, S, LOC, Y,0</b>	! select nodes at edge (Y=0)
<b>D, ALL, ALL</b>	! apply deflections (UX=UY=UZ=0) at (Y=0)
<b>NSEL, S, LOC, Y, b</b>	! select nodes at edge (Y=b)
<b>D, ALL, ALL</b>	! apply deflections (UX=UY=UZ=0) at (Y=b)

**Table (7): Modal analysis codes**

<b>/SOLU</b>	! start the preprocessing
<b>ANTYPE, MODAL, NEW</b>	! analysis type (MODAL analysis)
<b>MODOPT, QRDAMP,3</b>	! modal analysis is set
<b>MXPAND,3</b>	! three modes are expanded
<b>SOLVE</b>	! solution is started
<b>FINISH</b>	! finish

### **7.9 Modal Analysis Setup**

Setup and initiation of a modal analysis is very significant step. After preparing the model structure geometry, the modal analysis type is determined using ANTYPE command macro code. MODOPT command is defined to specify modals options analysis such as analysis type and number of degrees of freedom. MXPAND command is used to specify mode's number of modal analysis. Three modes are expanded in this analysis using MXPAND command. SOLVE command is used to start solution in

the ANSYS program macro code as given in Table (7). Three bending modes are detected from observing the deflections behavior of laminate plate structure. Using APDL ANSYS macro codes, three natural frequencies and three mode shapes are evaluated numerically.

## **8. Results and Discussion**

As shown in Figure (6) the natural frequency (Hz) of laminate plate varies with lamination angle ( $\theta$ ). The bending-stretching coupling due to the presence of  $B_{16}$  and  $B_{26}$  has the effect of lowering the frequencies. The coupling is maximum for two-layer plates, and it rapidly decreases with increasing number of layers. At  $\theta = 45$  the natural frequency of the two layers plate is about 40 percent lower than of eight-layer laminates. In figure (7) the relation between the natural frequency (Hz) and aspect ratio (a/b) for symmetric ((0/90)<sub>s</sub>, (0/90/0)<sub>s</sub> and (0/90/0/90)<sub>s</sub>.....etc) cross-ply laminates including simply support boundary condition SSSS. Four types of aspect ratios 1, 1.5, 2, and 2.5 are considered. For various aspect ratio, the specimen dimensions varied with fixing the plate thickness (h=1mm). An increase in aspect ratio (a/b) results a decrease in natural frequency of laminate plate, this is due to the decrease in stiffness matrix of composite. The natural frequency of two layers (0/90)<sub>s</sub> is about 37% lower than of (0/90/0/90/0/90)<sub>s</sub>. Figures (8,9) shows the relation between the aspect ratio (a/b) and natural frequency (Hz) for cross ply simply supported boundary conditions. A significant effect of coupling between bending and extension on the frequency of antisymmetric cross-ply laminates is found. It is observed that, the natural frequency decreases as the aspect ratio (a/b) increases. Figure (10) shows the natural frequency versus plate aspect ratio (a/b) for antisymmetric angle-ply laminates with under simply supported boundary conditions SSSS. It is noted that, in angle-ply laminates, the stiffnesses  $B_{16}$  and  $B_{26}$  go to zero as the number of layers in the laminate increases. That is why the natural frequency increases as number of layers increases and consequently the natural frequency decreases as aspect ratio increases. The effect of aspect ratio (a/b) on natural frequency for cross ply clamped boundary condition as shown in Figure (11). It is observed that as increasing the aspect ratio, the natural frequency of two layers decreases about 45% less than of four layers. The relation between the natural



frequency and modulus ratio of cross-ply and angle-ply laminates with simply support boundary condition as noted in Figures (12,13). It's observed that the modulus ratio proportional smoothly with the natural frequency. The symmetry laminate layer  $(-45/45)_s$  give us higher values than the antisymmetric layer  $(-45/45)_2$  due to the coupling between extension and bending is eliminated in symmetric layer. Mode shapes of vibration for laminate composite plate  $(0/90/0)$  with three degrees of freedom of frequency is shown in Figure (14). It is noted that, the fundamental mode occurs at first natural frequency. Table (8) illustrates a comparison of natural frequency (Hz) between analytical and finite element (ANSYS codes) results for cross-ply orientation angles and various aspect ratios  $(a/b)$  including simply support boundary conditions. It has been found that the maximum discrepancy between results is approximately equal to  $(0.1-0.2) \%$  for 1<sup>st</sup> frequency (fundamental frequency),  $(0.5-0.7) \%$  for 2<sup>nd</sup> frequency, and nearly  $(1.6) \%$  for 3<sup>rd</sup> frequency. A good verification results is obtained from this comparison. As given in Table (9), the higher range of natural frequency is obtained using fixed boundary conditions (CCCC) than those of simply supported (SSSS) boundary conditions. This discrepancy in results is due to restriction of four edges of the laminated plate as fixed boundary conditions. It is noted that, the clamped boundary conditions give frequency values double of that simply supported boundary conditions approximately.

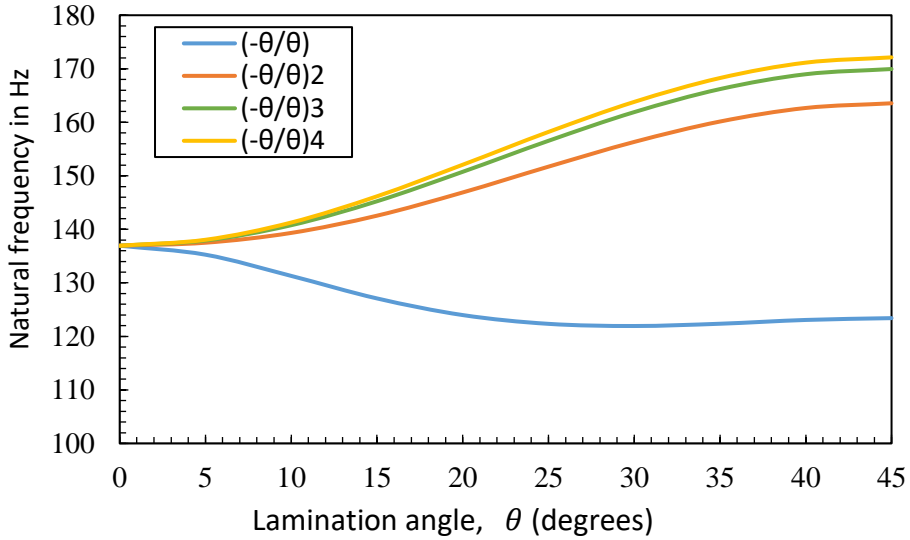


Figure (6): Fundamental Natural frequency versus lamination angle  $\theta$  degrees of antisymmetric angle-ply laminates and simply supported boundary

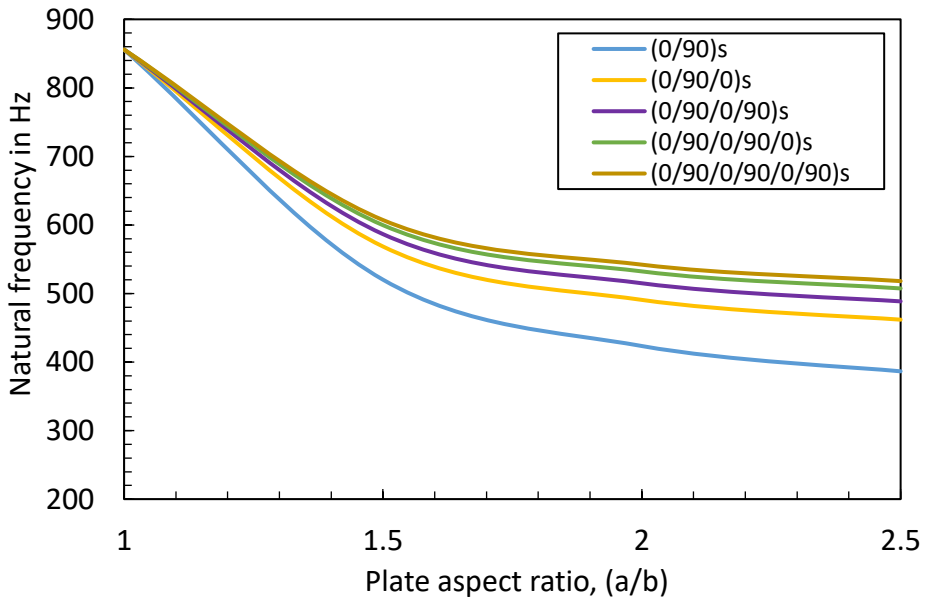


Figure (7): Fundamental Natural frequency versus plate aspect ratio  $(a/b)$  for symmetric cross-ply laminates and simply supported boundary

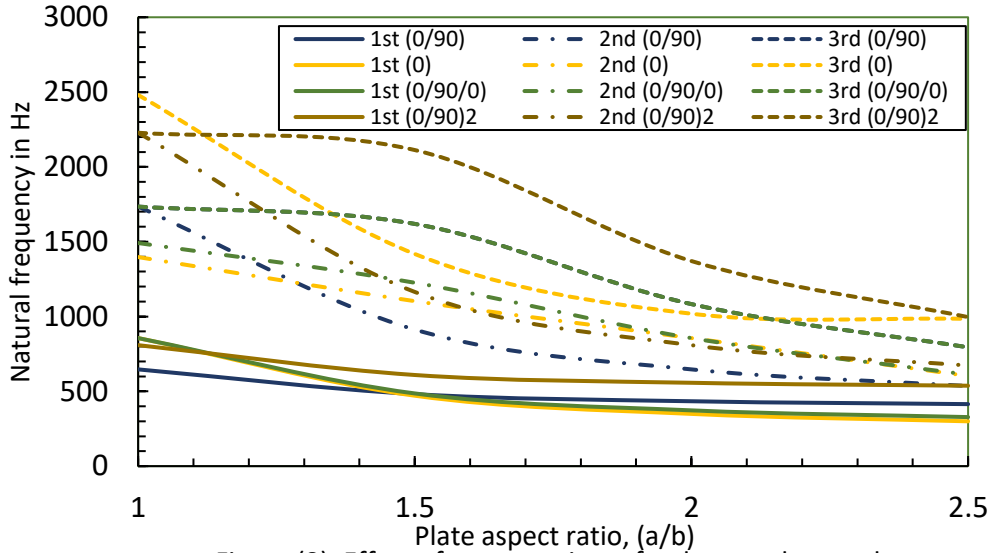


Figure (8): Effect of aspect ratio on fundamental natural frequency for cross ply simply supported boundary conditions

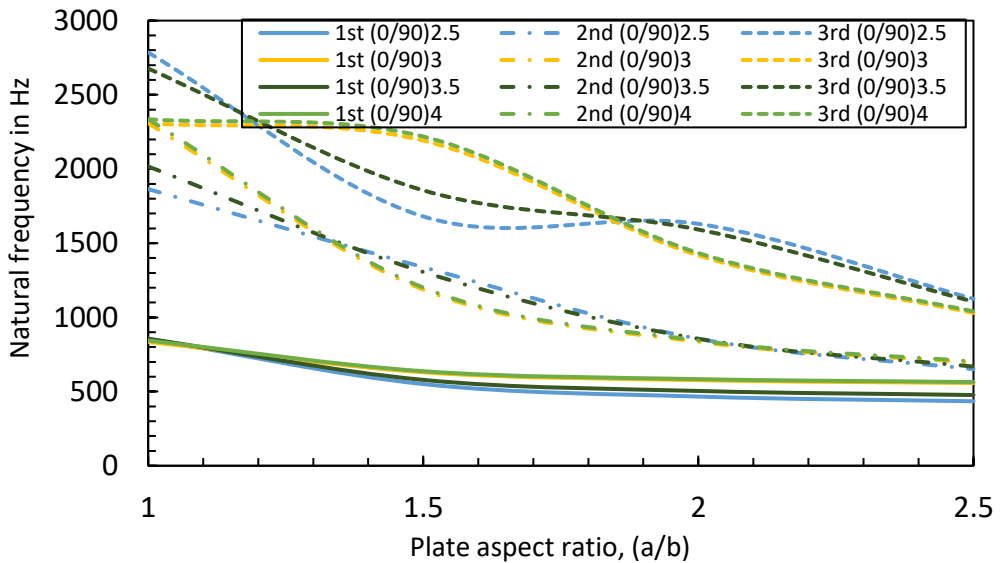


Figure (9): Effect of aspect ratio on fundamental natural frequency for cross ply simply supported boundary conditions

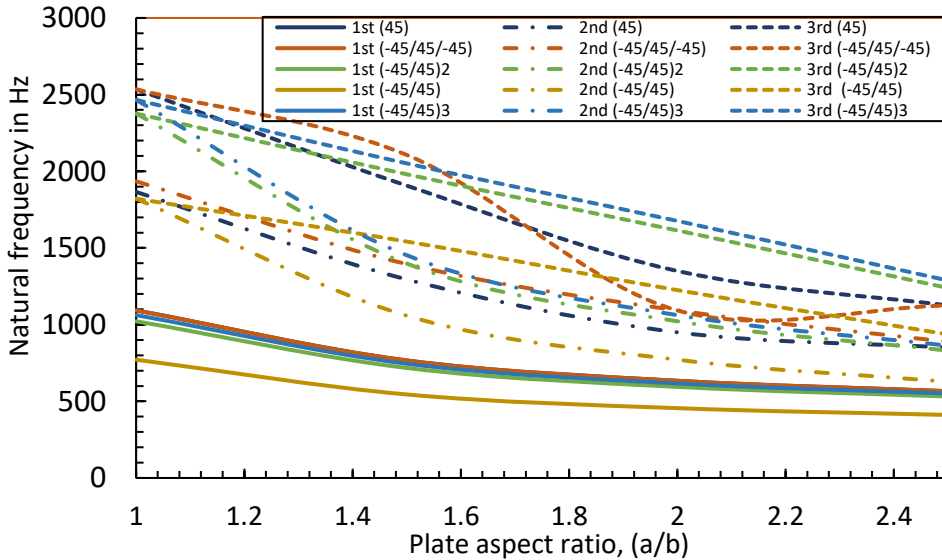


Figure (10): Effect of aspect ratio on fundamental natural frequency for angle ply simply supported boundary conditions

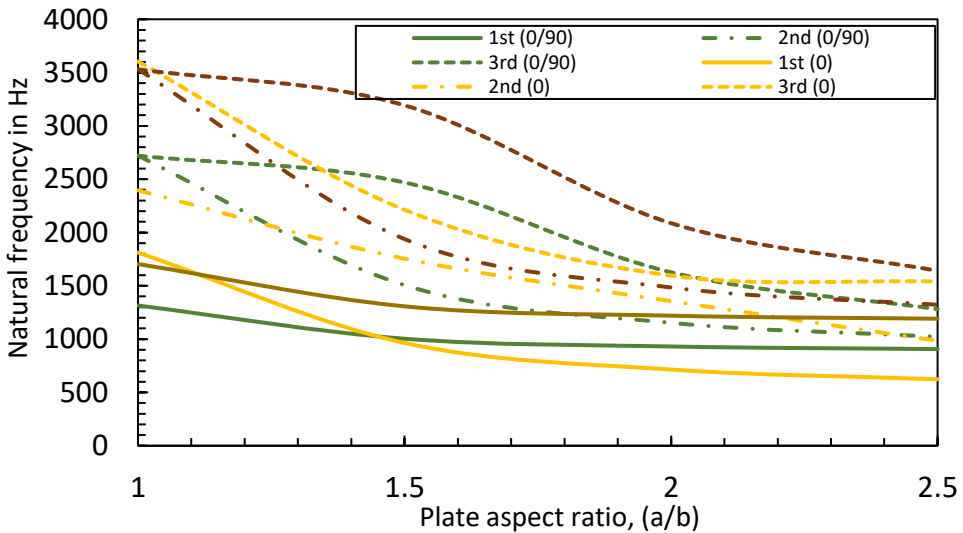


Figure (11): Effect of aspect ratio on fundamental natural frequency for cross-ply clamped boundary conditions

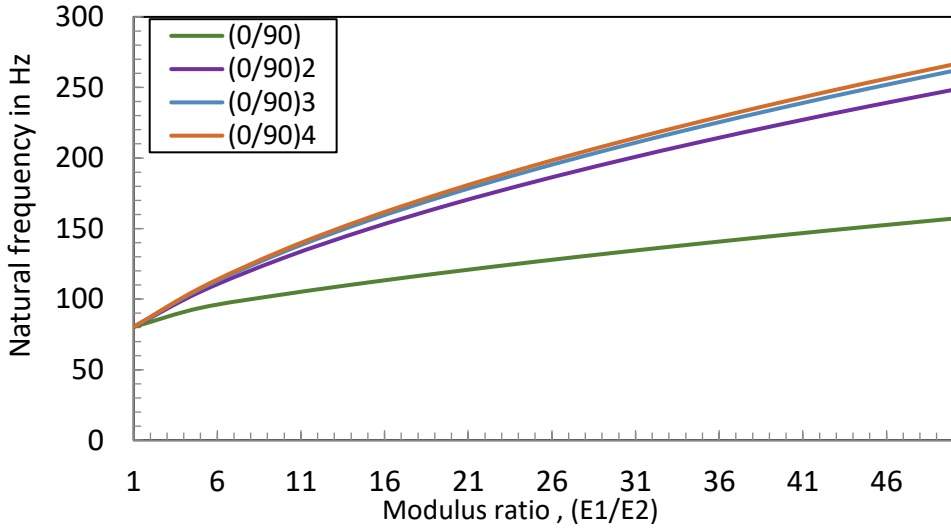


Figure (12): Fundamental frequency versus Modulus ratio for unsymmetric cross-ply laminates and simply supported boundary conditions

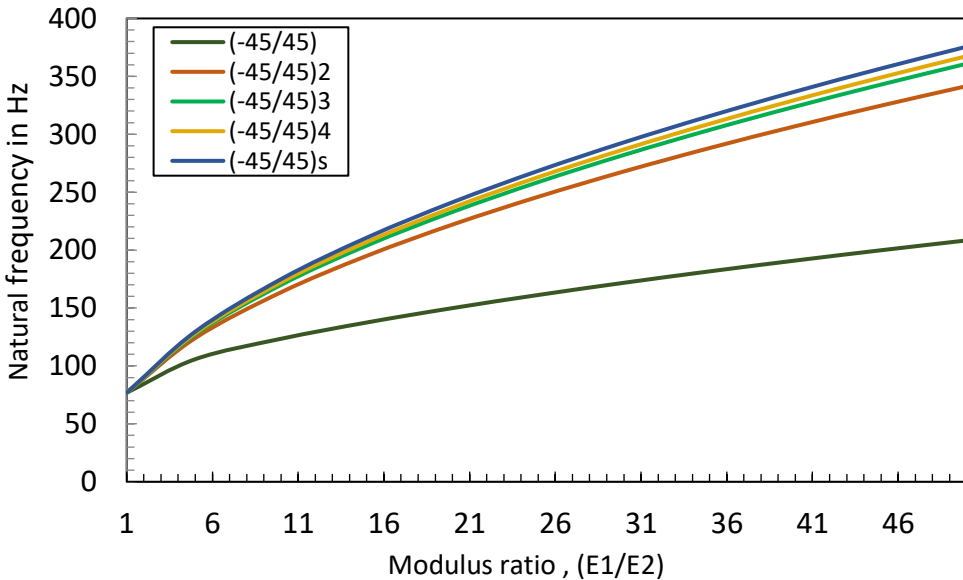
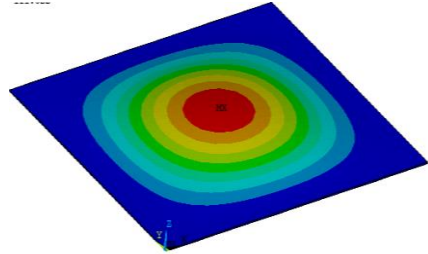
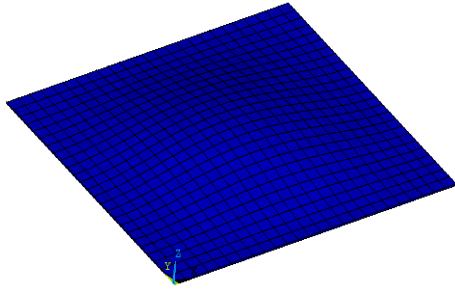
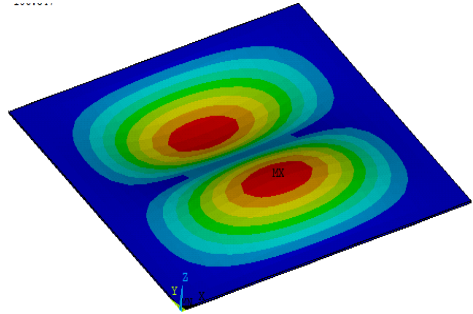
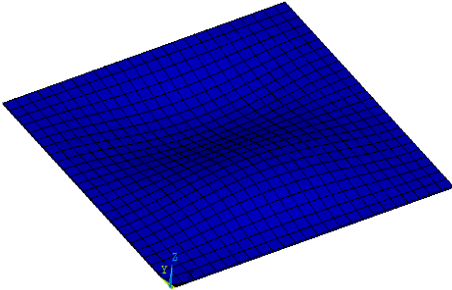


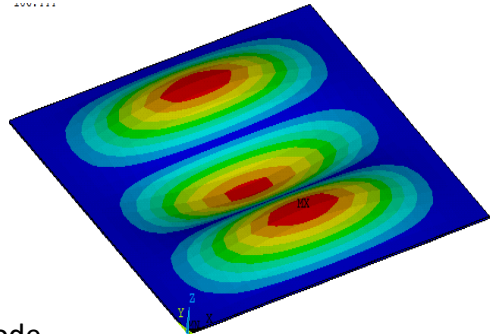
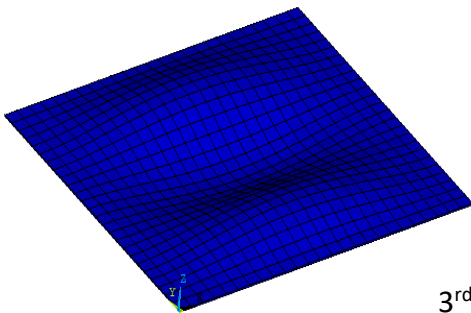
Figure (13): Fundamental Natural frequency versus modulus ratio (E1/E2) of unsymmetric angle-ply square laminates and simply supported boundary conditions



1<sup>st</sup> Fundamental mode



2<sup>nd</sup> Mode



3<sup>rd</sup> Mode

Figure (14): Mode shapes of vibration for laminate composite plate (0/90/0)



Table (8): Comparison of the natural frequencies in (Hz) between Analytical and Finite Element (APDL ANSYS Codes) Results for simply supported cross-ply orientation and various aspect ratios (a/b)

Aspect ratio (a/b)	1 <sup>st</sup> ANAL	1 <sup>st</sup> FEM	2 <sup>nd</sup> ANAL	2 <sup>nd</sup> FEM	3 <sup>rd</sup> ANAL	3 <sup>rd</sup> FEM
(0) Layers						
1	855.83	856.83	1397.1	1403.9	2480.6	2515.2
1.5	470.77	471.83	1102.7	1110.9	1418	1427.2
2	349.32	350.33	855.83	862.37	1018.7	1027.1
2.5	299.71	300.69	602.66	607.73	984.41	992.88
(0/90) Layers						
1	646.69	648.41	1734.5	1747.1	1734.5	1747.1
1.5	481.37	482.9	646.69	651.97	1618.9	1631.9
2	433.72	435.2	912.81	920.2	1083	1102.5
2.5	414.42	415.88	535.64	539.94	794.31	808.54
(0/90/0) Layers						
1	855.83	856.74	1491.1	1499.6	2756.9	2799.8
1.5	485.95	487.11	1225.6	1235.9	1404.1	1412.8
2	372.82	374.04	855.83	862.14	1151.6	1162.3
2.5	327.82	329.07	611.47	616.52	1121.7	1132.5
(0/90/0/90) = (0/90)2 Layers						
1	808.63	810.08	2228.1	2241.7	2228.1	2241.7
1.5	609.28	610.71	1164.2	1156.7	2112.5	2127
2	557.12	558.6	808.63	813.6	1370.8	1392.6
2.5	537.5	539.02	672.56	676.34	996.04	1010.9
(0/90/0/90/0) Layers						
1	855.83	856.9	1864.5	1876.3	2785.1	2797.3
1.5	550.64	551.95	1338.3	1346.7	1679.9	1693.3
2	466.19	467.62	855.83	861.66	1630.2	1644.1
2.5	434.61	436.12	650.56	655.1	1125	1143.3
(0/90/0/90/0/90) = (0/90)3 Layers						
1	835.18	836.49	2307.9	2321.4	2307.9	2321.4
1.5	630.13	631.47	1187.7	1195.2	2191.8	2206.3
2	577.08	578.51	835.18	840.02	1417.7	1439.7
2.5	557.32	558.8	694.96	698.6	1029.1	1043.9
(0/90/0/90/0/90/0/90) = (0/90)4 Layers						
1	844.27	845.53	2335.3	2348.6	2335.3	2348.6
1.5	637.26	638.58	1200.9	1208.3	2218.9	2233.3
2	583.91	585.31	844.27	849.06	1433.7	1455.8
2.5	564.09	565.56	702.64	706.22	1040.4	1055.2

Table (9): Frequencies in (Hz) for simply supported (SSSS) and clamped boundary conditions (CCCC) using finite element (ANSYS) results for cross-ply orientation and various aspect ratios (a/b)

Aspect ratio (a/b)	1 <sup>st</sup> CCCC	1 <sup>st</sup> SSSS	2 <sup>nd</sup> CCCC	2 <sup>nd</sup> SSSS	3 <sup>rd</sup> CCCC	3 <sup>rd</sup> SSSS
(0) Layers						
1	1811.3	856.83	2395.8	1403.9	3604.4	2515.2
1.5	964.21	471.83	1754.4	1110.9	2213.2	1427.2
2	714.65	350.33	1356.3	862.37	1596.2	1027.1
2.5	623.05	300.69	985.02	607.73	1540.6	992.88
(0/90) Layers						
1	1314.8	648.41	2720.2	1747.1	2720.2	1747.1
1.5	1003.8	482.9	1506.5	651.97	2469.2	1631.9
2	931.15	435.2	1152.8	920.2	1627.0	1102.5
2.5	906.43	415.88	1022.2	539.94	1281.5	808.54
(0/90/0) Layers						
1	1810.6	856.74	2524.5	1499.6	3971.0	2799.8
1.5	1002.8	487.11	1947.1	1235.9	2201.3	1412.8
2	774.41	374.04	1374.5	862.14	1809.5	1162.3
2.5	693.35	329.07	1023.6	616.52	1667.8	1132.5
(0/90/0/90) = (0/90)2 Layers						
1	1704.0	810.08	3531.0	2241.7	3531.0	2241.7
1.5	1307.6	610.71	1938.7	1156.7	3192.6	2127
2	1219.7	558.6	1484.3	813.6	2087.4	1392.6
2.5	1191.4	539.02	1322.2	676.34	1640.5	1010.9
(0/90/0/90/0) Layers						
1	1813.2	856.9	3044.9	1876.3	4352.0	2797.3
1.5	1163.6	551.95	2152.2	1346.7	2651.2	1693.3
2	1002.9	467.62	1454.4	861.66	2362.5	1644.1
2.5	950.11	436.12	1181.2	655.1	1697.4	1143.3
(0/90/0/90/0/90) = (0/90)3 Layers						
1	1766.4	836.49	3660.7	2321.4	3660.7	2321.4
1.5	1356.2	631.47	2008.2	1195.2	3308.6	2206.3
2	1265.7	578.51	1537.6	840.02	2161.6	1439.7
2.5	1236.8	558.8	1370.3	698.6	1698.3	1043.9
(0/90/0/90/0/90/0/90) = (0/90)4 Layers						
1	1787.7	845.53	3705.0	2348.6	3705.0	2348.6
1.5	1372.8	638.58	2032.0	1208.3	3348.2	2233.3
2	1281.5	585.31	1555.8	849.06	2186.9	1455.8
2.5	1252.3	565.56	1386.8	706.22	1718.1	1055.2

## **Conclusion**

A new solution method has been presented for free vibration analysis of rectangular laminated composite plate. Strength of the method is the possibility of analyzing plates with general stacking sequences including symmetry, unsymmetrical laminates with elastic couplings. Furthermore, any boundary conditions corresponding to clamped and simply supported can be considered. The proposed method has been applied to several different cases and the conclusions can be mentioned as bellow:

- 1- It's noted that the fundamental natural frequency decreases as the aspect ratio increases for all type cases.
- 2- It's found that the fundamental natural frequency increases when modulus ratio ( $E1/E2$ ) increases.
- 3- It's obtained that any increase in lamination angle ( $\theta$ ) leads to increase in the fundamental natural frequency.
- 4- A good agreement is found between analytical and finite element results for different aspect ratio ( $a/b$ ).
- 5- The fundamental frequencies of free vibration of clamped plates are higher than from simply support plate.
- 6- It's concluded that any increase in ply layers will results an increase in fundamental natural frequency.

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## شيكاركردي له ره له ري پليتي كومبوزيت به به كارهيناني ريكاى تيوري و كودي نه

### نسز

**پوخته:** مه به ست له م كاره بريتيه له ليكولينه وه ي تيوري (بيردوزى كلاسيكى) و ريكاى (APDL) بؤ پليتي كومبوزيتى تويكراو ليكولينه وه ي له رينه وه ي نازادى له كه ل دوو شيوازي پا لبشتى كرتنى ليواره كان كه پيك ديت له هه ريه كه له پا لبشتى ساده و كيره له م كاره به كارهاتوه . هاوكيشه ي جوله كه به كارديت بؤ فه رمانره واييكردى كرده ي پيشبيني شيوازي له رينه وه ي جينى خاج و جينى كوشه ي شيكاركراون به ريكاى تيوري به به كارهيناني سوفتويرى ماتلاب . كردارى بايه خدار بؤ هه ريه كه له زماره ي جينه كانى فاييه ر , ديمه نى ريزه يى , لارى فاييه ر له كه ل هاوكولكه ي ريزه يى له سه ر له ره له ره ي سروشتى به هه ريه كه له ريكاى تيوري و نزيكراوه يى هه زماركرا . كونجانيكى باشى نه نجامه كانمان ده ست كه وت له نيوان هه ردوو ريكاى تيوري و نزيكراوه يى به كارهاتوو .

## التحليل الاهتزازي لطبقات الصفائح المركبة باستخدام الدراسة التحليلية وكودات الانسز

### المخلص:

الهدف من هذا العمل هو تحقيق التحليل النظري و التحليل العددي لطبقات الصفائح المركبة . تم دراسة الترددات الطبيعية للصفائح المركبة مع أخذ حالتين تثبيت حدود الطبقة و هي كل من حالة التثبيت الجزئي و حالة التثبيت الكلي . ان معادلة الحركة التى تتحكم في تخمين سلوك الاهتزاز لطبقات الزوايا المتعامدة و طبقات الزوايا المائلة لصفائح مركبة تم تحليله نظريا باستخدام برنامج ماتلاب . ان التأثير الواضح لعدد من العوامل لكل من عدد الصفائح , زاوية الالياف , نسبة الطول الى العرض و نسبة معامل المرونة وتأثيرها على الاهتزاز الحر تم حسابه تحليليا و عدديا . وتم حساب الحالة العددية بواسطة برنامج (APDL ANSYS) . لقد كان تأثير حالة التثبيت واضح على الاهتزاز الحر للصفائح المركبة بسبب قيود الحركة على حواف البلية . لقد تم الحصول على مقارنة ونتائج جيدة بين النتائج التحليلية و النتائج العددية .