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Proposed Hybrid Method for Wavelet Shrinkage with Robust Multiple Linear Regression Model (With Simulation Study)

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ABSTRACT

This study compares the proposed hybrid method (wavelet robust M-estimation) to the traditional method (wavelet ordinary least square) when there are de-noising or outlier problems for estimating multiple linear regression models using the statistical criterion root mean square error (RMSE). According to simulated and real data, the proposed hybrid method (wavelet robust M-estimation) is better than the classical method (Wavelet Ordinary Least Square) and more accurate. The root mean square error of the proposed hybrid method (wavelet robust M-estimation) is less than the Wavelet Ordinary Least Square. Therefore, it is recommended to use the hybrid proposed method to reduce the problem of outliers and de-noise data.

1. Introduction

Wavelet regression is a technique for reducing noise in a sampled function that has been contaminated by noise. The wavelet decomposition coefficients, which mainly represent noise, are thresholded to achieve this. The wavelet theory is a current and important theory that has a wide range of applications in both theoretical and applied

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disciplines. Wavelet shrinkage estimation, based on a thresholding parameter, has lately become a powerful mathematical technique for de-noising function estimates, and the choice of this threshold dictates, to a large extent, the efficiency and success of de-noising. Signal structure will be lost in general if the threshold setting is set too high. If it is set too low, though, noise will be included in the estimate. In regression modeling, identifying outliers or contaminated observations is a critical step. Diagnostic approaches for detecting a single outlier or contaminated observation have been widely developed, and most statistical packages include them. Wavelets have a formal history that dates back to the early 1980s, when they were created to investigate seismic signals (Goupillaud, Grossmann, and Morlet, 1984). During the rest of the 1980s, wavelet analysis remained popular among a small, mostly mathematical population, with only a few scientific papers published each year. (Danoho and Johnstone 1994 a) involved wavelet analysis in statistical fields for recovering noise-contaminated signals using orthonormal bases of compactly supported wavelets provided by (Daubechies I., 1988) by improving the non-linear in wavelets through thresholding. In (2001) Sardy,S. et al , proposed a robust wavelet-based estimator using a robust loss function. The wavelet estimators of the nonparametric regression function based on various thresholds under the mixture prior distribution are obtained in (2020) (Afshari, M. et al.). The researcher was interested in constructing an effective model with multiple-linear model parameters as well as diagnosing outlier values by evaluating the wavelet with the approach of ordinary least squares (OLS) and robust M- estimation. As a result, the outlier values are revealed and treated, and a multi-linear model with wave shrinkage is estimated, which involves the wave functions and the use of a method for estimating the level of threshold, including soft threshold, followed by a comparison of the estimates' efficiency with an ordinary and robust method fortified with some wavelet filters.

2: Theoretical part

2.1: Linear regression model

Linear regression is a statistical method for modeling the relationship between one or more explanatory (independent) factors and a response (dependent) variable. It's

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one of the most important statistical tools, with uses in a variety of fields (Montgomery, et al. 2012).

2.1.1 Multiple Linear regression model

Regression analysis is a statistical method for investigating and fitting an unknown model for quantifying relationships between observable variables (Hisham M. Almongy & Ehab M. Almetwally, 2017).

Considering the multiple linear regression model with n observations and p independent variables, it can be defined as, $\mathrm{Y}_{(n*1)} = \mathrm{X}_{(n*p)} + \beta_{(p*1)} + \varepsilon_{(n*1)}$ where Y_{p*1} is a response vector, X_{n*p} a non-stochastic input matrix , β_{p*1} an unknown vector of coefficients and ε_{n*1} an error vector distributed normality with $E(\varepsilon) = 0$ and $\mathrm{var}(\varepsilon) = \sigma^2 I_n$. We use the least squares method to estimate the parameters, which minimizes the sum of squared deviations of the observed and fitted responses, also known as the residual sum of squares.

$$\sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon'\varepsilon = (Y - X\beta)'(Y - X\beta) \qquad ..... (1)$$

The ordinary least squares estimator $\hat{\beta}_{OLS} = (x'x)^{-1} x'y$ is obtained by minimizing the error sum of square. With $\mathrm{E}(\hat{\beta}) = \beta$ and $\mathrm{var}(\hat{\beta}) = (X'X)^{-1}\sigma^2 I$ Then the vector of (n) least square residuals is $e_i = (Y_i - \hat{Y}_i)$ the residual mean square estimator of $\hat{\sigma}^2$ is

$$\hat{\sigma}^2 = \frac{\varepsilon'\varepsilon}{n-p-1} = \frac{(Y - X\beta)'(Y - X\beta)}{n-p-1} \qquad .... (2)$$

2.2 Outlier

The high or low value of the observation is defined as the outlier value, which is described by the data's behavior. That departs from the behavior of the rest of the data, or in other words is an observation that is inconsistent in value with the rest of the views is away from the data center. The occurrence of outliers in the data from the regression model.

It can occur in the dependent variable (the response variable), in which case it is referred to as an "outlier," or in the explanatory variables, in which case it is referred to as (leverage points), which can be of two types: good lift points that do not affect the estimation of the regression equation, or poor lift points that do affect the parameters of the model. Outlier values may exist in the dependent variable as well as in the explanatory factors, compounding the estimation challenge. The method of least squares, for example, is inefficient since the model's errors do not follow a normal distribution, necessitating the employment of additional approaches to avoid these issues and produce efficient parameter estimations (Rousseeuw, P.J., and A. Leroy, 1987).

Data contamination is unavoidable in data analysis, necessitating the employment of robust statistical approaches (kanamori, T., and Fujisawa, H., 2005). the data comes from two different sorts of distributions. The first is called "Distribution Basic," and it produces solid data. Outliers are created via the second category, referred to as "Distribution Contaminating." This is understandable mathematically. If denotes the probability of the contaminant distribution and is the probability density function of the basic distribution, then the distribution of any observations will be (Hawkins, 1980):

$$f(x) = (1-p) * f_1(x) + p * f_2(x) \qquad .... \ (3)$$

When normal data points are used, the normal distribution $f_1(x)$ is used. When contamination, mixed distributions, or heavy tail distributions are employed, $f_2(x)$ is used. And (p) is the contamination ratio.

2.3 : Robust (M- Estimation)

M-estimation, proposed by Huber, is the most frequent generic approach to robust regression (1964). The term "M"-estimation refers to a class of estimators that can be thought of as a generalization of maximum-likelihood estimation (Fox, John, & Weisberg, Sanford, 2018). The M-estimation approach uses iteratively reweighted least squares to solve this system (IRLS). Under this condition the likelihood functions for β and σ is:

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$$L(\beta,\sigma)=\frac{1}{\sigma^n}\prod_{i=1}^{n} f\left(\frac{y_i - x_i{}'\beta}{\sigma}\right) \quad \text{..... (4)}$$

Where

$x_i = (1, x_{i1}, x_{i2}, \ldots\ldots, x_{ip-1})$. By replacing the ordinary least squares criterion with a robust criterion, M-estimator of β is:

$$\hat{\beta}_M = \min_{\beta} \sum_{i=1}^{n} \rho\left(\frac{y_i - x_i{}'\beta}{\hat{\sigma}_M}\right) \quad ; \hat{\sigma}_M = \frac{Med\left|e_i - Med(e_i)\right|}{0.6745} \quad \text{.... (5)}$$

Where (e_i) denotes the(i-th) residual. We obtain the following normal equations:

$$\sum_{i=1}^{n} x_{ij}\psi\left(\frac{y_i - x_i{}'\beta}{\hat{\sigma}_M}\right) = 0 \quad for \;\; j = 0 \;,\; 1 \;,\; 2, \ldots, p-1$$

Where $x_{i0} = 1$ *and* $\psi(.)$ the first derivative is function of $\rho(.)$ and is called the influence function. To solve the M-estimates nonlinear normal equations, the iteratively reweighted least squares (IRLS) approach was utilized. The iterative algorithm that follows is (Ruckstuhl, 2014):

1. Start with the OLS estimate as an initial estimate of $\hat{\beta}$ and then estimate of $\hat{\sigma}_M$

2.Compute the weights, such as w_i .

3. Compute a new estimate of β using Eq. (5).

4. Keep repeating steps 2 and 3 until the algorithm has converged. Finally, the M-formula estimator's

$$\hat{\beta}_M = (x'wx)^{-1} x'wy \;\; ; w = diag(w_i) \quad \text{..... (6)}$$

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Table (A) Displays objective and weight function used in robust regression (K. MacTavish and T. D. Barfoot. 2015).

Table A: Objective and weight functions

Method	Objective function	Weight function	Constant
Cauchy	$\frac{c^2}{2}\log\left(1+\left(\frac{u_i}{c}\right)^2\right)$	$\frac{1}{1+\left(\frac{u_i}{c}\right)^2}$	C=2.385

The value (u) in the weight functions is $u_i = \frac{e_i}{\hat{\sigma}_M}$.

2.4: Wavelet Analysis

The basic purpose of applying filters in linear regression analysis is to eliminate noise. Recent advancements in regression model analysis have centered on the use of wavelet filters rather than traditional filters, which are, of course, superior and more efficient (Hamad.A.S, 2010).

Researchers (Morris J. M. and Peravali R., 1999) used discrete Wavelet Transform coefficients as filters for contaminating observation. A filter can be thought of as a type of operator. ℓ_2 (Discrete form of $L^2(R)$) in to itself, a noise-contaminated signal is passed through a filter to isolate the signal or extract the noise. An observed (discrete) observation g is generally represented by a sequence $\{g_i\}$ $i \in z$, assuming $g \in \ell^2(z)$.

A filter (A) can be presented by a $\ell^2(z)$ sequence $\{a_i\} i \in z$. A discrete convolution of the filter sequence with the observation is used in the filtering process. The filter (A) is applied to the observation (g) as follows:

$$(Ag)_i = \sum_{\ell \in z} a_{\ell-2i} g\ell$$

Creating a new observation with the index (i), which ranges over (z).

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The DWT is based on filter H and G defined respectively by $\{h_i\}i \in z$ and $\{g_i\}i \in z$ the filters that were developed from the multiresolution analysis must meet the following criteria:

1- The stability of coefficient h_i .

$$\sum_{i=0}^{n-1} h_i = 0 \qquad ....(7)$$

2- The requirement of wavelet expansion convergence requires the condition.

$$\sum_{i=0}^{n-1} (-1)^i r^m h_i = 0 \qquad m = 0,1,\ldots,\frac{n}{2}-1 \qquad .....(\ 8)$$

3- The orthogonally of wavelets requires the condition

$$\sum_{i=0}^{n-1} h_i h_{i+2m} = 0 \qquad m = 0,1,\ldots,\frac{n}{2}-1 \qquad ......(\ 9)$$

4- Finally, if an orthogonal scaling function is desired.

$$\sum_{i=0}^{n-1} h_i^2 = 0 \qquad ....(10)$$

2.4.1: Discrete Wavelet Transform

A Discrete Wavelet Transform is a set of coefficients in the time and frequency domain that summarize the information of all observations with a reduced number. Discrete Wavelet Transformation (DWT) is employed in a wide range of applications, especially when data contains contaminants or noise. (DWT) uses scaled and shifted versions of a compact supported basis function to decompose a signal (Walker J.S., 1999)and (Abramovich F., et al., 2000)Given a vector of a signal (X) consisting of 2^j observation. The (DWT) of X is

$$W = wX \qquad ..... (11)$$

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Where W is a ($n*1$) vector comprising both discrete scaling and wavelet coefficients. The vector of wavelet coefficients can by organized into $j+1$ vectors.

$$W = [W1, W2, \ldots, Wj_0, Vj_0]^T$$

Where Wj is a length $N_j = N/2^j$ vector of wavelet coefficients (Details) associated with changes on a scale of length $\lambda_j = 2^{j-1}$ symbol as CD, and(V_{jo})is a length $N_{j0} = N/2^j$ vector of scaling coefficients (approximation or smoothing) associated with average on a scale of length $\lambda_{J_0} = 2^{J_0}$ symbol as CA, and w is an orthonormal $N*N$ matrix associated with the orthonormal wavelet basis chosen(Antoniadis,A.,2007) (Gencay,R.,et al.,2002).

The approximation coefficients are separated into bands after each DWT using the same filter as previously, resulting in the details being attached to the details of the most recent decomposition, and the signal may be reconstructed from the de-noise signal using the inverse transform at each level.

$$X = W w^T = \sum_{j=1}^{j_0} W_j^T W_j + V_{j_0}^T V_{j_0} \qquad \text{.... (12)}$$

2.4.2: Wavelet Shrinkage

The noise or contaminant removal of the observation is one of the most common applications applied to the view following its analysis by wavelet transformation. The scientists discovered that the noise produced after the conversion has a lower frequency than the original observation, utilizing Discrete Wavelet Transformation (DWT). Shrinkage is typically used to minimize the risk level or reduce the noise or outlier by depending on threshold, which is any acceptable frequency threshold setting that cancels the noise coefficients while maintaining the original observation coefficients. This is the simplest non-linear reduction of the wavelet coefficients introduced by (Donoho & Johnston, 1994a), which obtains a summary of the significant transformation coefficients that pass the threshold cut as a test for them,

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so that the coefficients are zero if their absolute value is less than a specific threshold cut level, attempts to recover a signal $g(t)$ from noisy an observation $x(i)$

$$x(i) = g(i) + v(i) \qquad i = 0,1,2,\dots,N-1 \qquad \dots. \ (13)$$

As a result, the following essential stages, which represent a summary of the wavelet shrinkage approach, must be taken:

1-The DWT transforms the data into a distinct representation known as wavelet coefficients. W, an orthogonal matrix, is multiplied by them.
2- A thresholding rule is used to modify the wavelet coefficients. The fundamental principle of wave shrink is to reduce the number of coefficients.
3- To estimate the signal, the changed coefficients are subjected to the inverse discrete wavelet transformation (IDWT).

As a result, the three-step wavelet shrinkage process can be illustrated as follows:

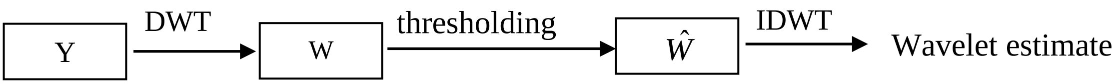

Diagram (1): steps of wavelet shrinkage

2.5: Proposed method

The proposed hybrid method uses wavelet shrinkage and robust M-estimation to estimate parameters of a multiple linear regression model, then uses the outputs to find the inverse of the Discrete Wavelet Transformation (DWT) and filter data, and then uses this data modified to estimate parameters of a multiple linear regression model using (Wavelet robust M-estimation, wavelet OLS) and calculating RMSE and comparing it to the classical method.
By shrinking the detail coefficients, which we can get by re-covering the original observations and splitting them into two components using wavelets, one of the types of thresholds, such as hard or soft, is usually used to isolate outliers or contaminants from the values of observations of the dependent variable. The first is

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the sum of the coefficient details, and the second is the smoothing parameters calculated using Multiple Re-Resolution Analysis, which is as follows:

$$Y = w^T W = \sum_{j=1}^{J_0} w_j^T W_j + v_{J_0}^T V_{J_0} \qquad ..... (14)$$

Applying a soft threshold to the DWT coefficients and returning the remaining coefficients to the vector elements (W'), the DWT coefficients of the modified wavelet, typically indicated by (w), can be obtained, allowing us to re-cover the observations of the treating dependent variable, i.e.

$$\tilde{Y} = w^T W' \qquad .....(15)$$

We receive the values of (observations of the processed dependent variable) based on the wavelet matrix (db-7) and (bior-1.1), which will be employed with the independent variable in estimating the parameters of multiple linear regression i.e.

$$\underline{\hat{\beta}}_{wavelet\ M-estimation} = (X'WX)^{-1} X'W \tilde{Y} \qquad ....(16)$$

Finally, as illustrated in diagram (2), the approaches used to analyze multiple linear regression and compare their performance in the presence of outlier and noise values (wavelet robust M-estimation and wavelet OLS) will be describe:

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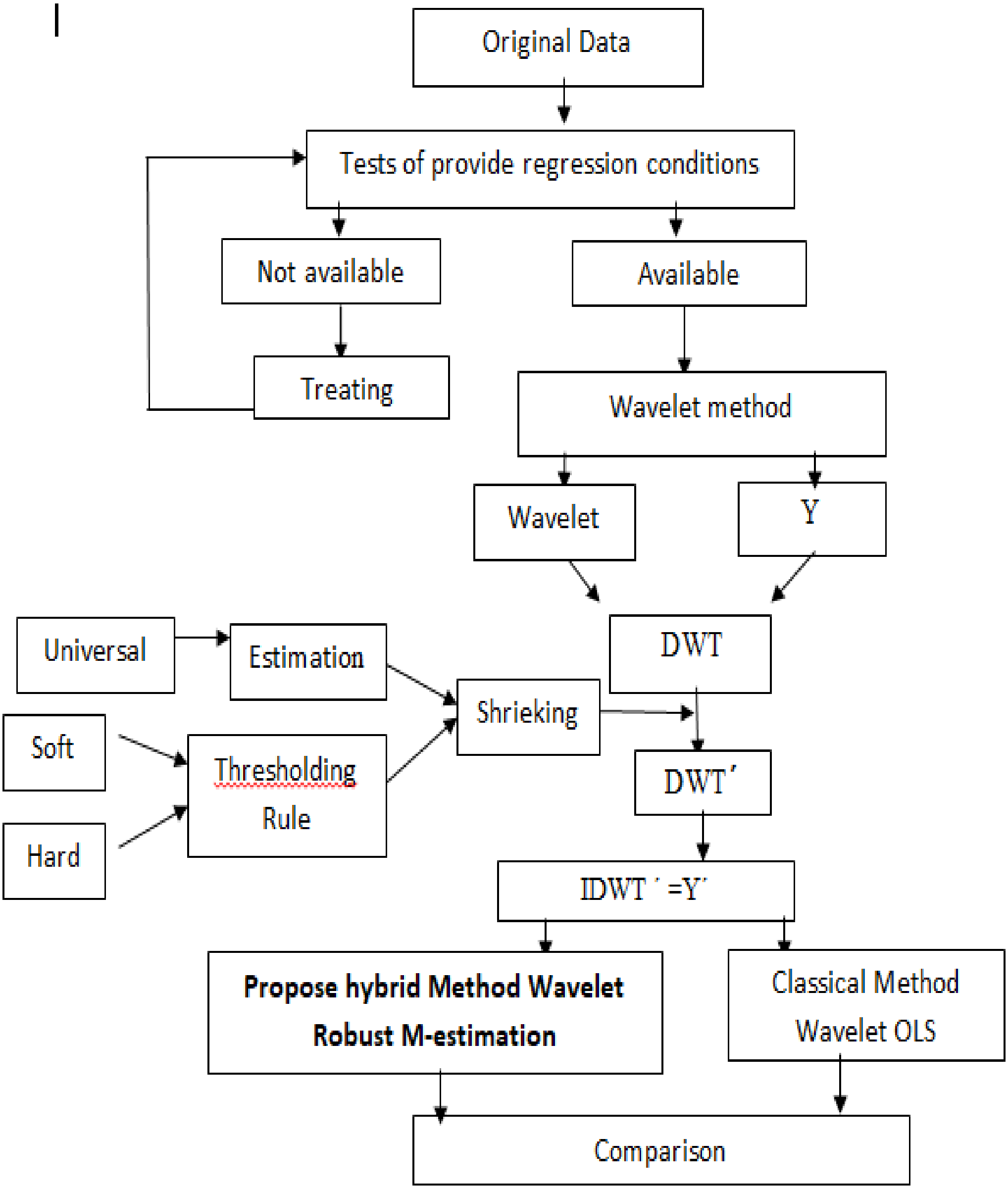

Diagram (2): proposed and classical method

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3: Application Part

A practical comparison of the proposed hybrid method (wavelet robust M-estimation) with the classical method (wavelet OLS) is shown in this section. The comparison was conducted by measuring relative efficiency, which indicates the root mean square error (RMSE), and then reviewing the most essential method for reducing data noise.

3.1: Description and analysis of simulation experiment

Different levels of the following factors were utilized to implement the simulation experiment sample sizes n, Where three sample sizes were used, namely $2^6 = 64$, $2^7 = 128$, $2^8 = 256$ The sample size here should be $n = 2^j$ whereas (j) a positive integer.

When the number of parameters (k) is equal to (2, 5, and 10) and the (e_i) vector is contaminated (10%) without changing the explanatory variables, the contaminated values can lead to outliers. The original (e_i) values are derived from a standard normal distribution, and a Cauchy distribution is constructed. These values, when calculated using this method (3), will invariably result in outliers and taint the data. The explanatory variables are independent of a standard normal distribution .A comparison was made between the methods used in the estimation represented by the method wavelet robust M-estimation with wavelet OLS for the frequency of (1000) replications of the assumed regression model and for each of the cases shown in the tables (1, 2, 3), and parameters for the default model can now be defined. The relative efficiency, which represents the root mean square error, was used to do the comparison (RMSE).

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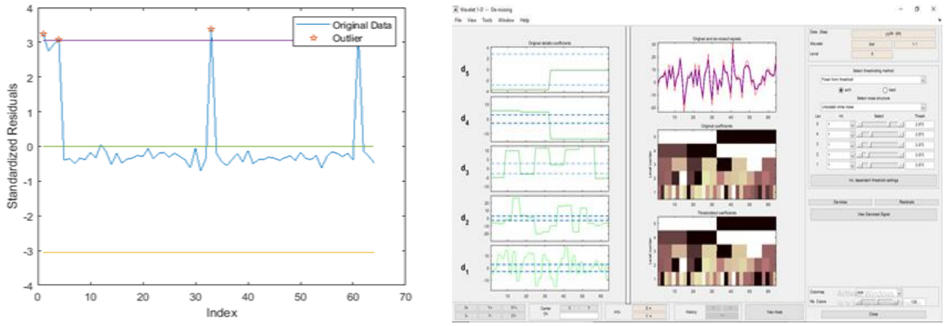

Figure 1: wavelet analysis Shows the noise and outlier plot where (10%) contaminate for (n=64)

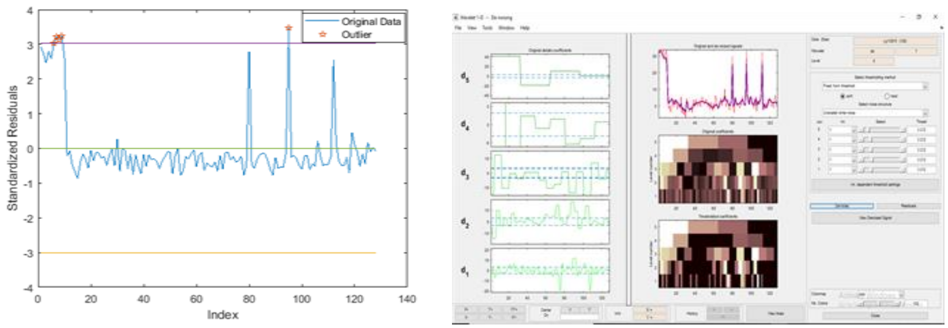

Figure 2: wavelet analysis Shows the noise and outlier plot where (10%) contaminate for (n=128)

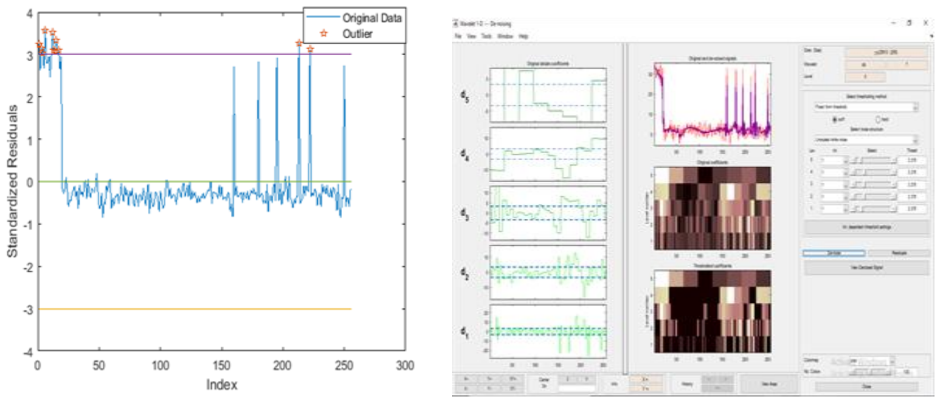

Figure 3: wavelet analysis Shows the noise and outlier plot where (10%) contaminate for (n=256)

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Table 1: the Average of RMSE values for different estimation method when (k=2)

Method Estimation	RMSE					
	σ = 1			σ = 5		
	n=64	n=128	n=256	n=64	n=128	n=256
Wavelet OLS-bior1.1	7.7687	8.4670	8.1351	8.3790	9.0488	8.7481
WaveletOLS-db7	7.7373	8.3653	8.1303	8.3527	8.9935	8.8093
Wavelet robust M-estimation -bior1.1	**3.0571**	**2.5823**	**2.0748**	**5.2207**	**5.0379**	**4.8142**
Wavelet robust M-estimation -db7	3.5260	2.8314	2.3070	5.4729	5.1894	4.9173

Table 2: the Average of (RMSE) values for different estimation method when (k=5)

Method Estimation	RMSE					
	σ = 1			σ = 5		
	n=64	n=128	n=256	n=64	n=128	n=256
Wavelet OLS-bior1.1	8.4623	8.8725	8.5862	9.0725	9.6460	9.4261
WaveletOLS-db7	8.4763	8.7945	8.6102	8.9270	9.5975	9.4597
Wavelet robust M-estimation -bior1.1	**5.6686**	**4.9424**	**4.1271**	**8.3199**	**8.1398**	**7.4114**
Wavelet robust M-estimation -db7	6.0938	5.1789	4.4656	8.2528	8.2013	7.5238

Table 3: the Average of (RMSE) values for different estimation method when (k=10)

Method Estimation	RMSE					
	σ = 1			σ = 5		
	n=64	n=128	n=256	n=64	n=128	n=256
Wavelet OLS-bior1.1	8.6021	9.0276	8.7741	9.0550	9.4821	9.2288
WaveletOLS-db7	8.6539	8.9751	8.8141	9.0925	9.4255	9.2593
Wavelet robust M-estimation -bior-1.1	**7.4384**	**6.9535**	**5.9825**	**7.1488**	**6.6452**	**6.1288**
Wavelet robust M-estimation -db7	7.6877	7.0627	6.1846	7.4860	6.8225	6.3233

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3.2: The Real data

The data collection includes seven variables for a total of 32 countries. These data, according to Gunst and Mason (1980, Appendix A), are a subset of a broader data set (data set 41 of Loether et al., 1974), so we use the same terminology.

X_1: Infant deaths per 1000 live births.

X_2 : Number of inhabitants per physician.

X_3: Population per square kilometer.

X_4 : Population per 1000 hectares of agricultural land.

X_5: Percentage literate of population aged 15 years and over.

X_6: Number of students enrolled in higher education per 100,000 populations

Y : Gross national product per capita, 1957 U.S. dollars.

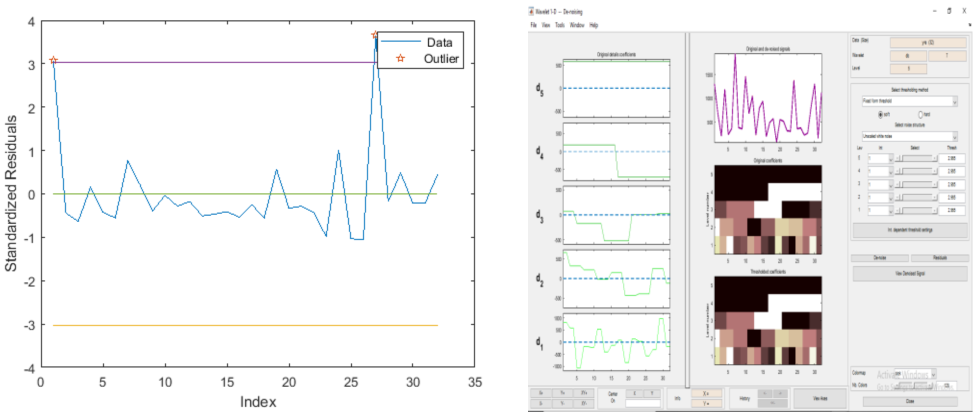

Figure 4: wavelet analysis shows the noise and outlier plot for real data

Table 4: (RMSE) values for different estimation method

Method Estimation	RMSE
Wavelet OLS-bior-1.1	676.0195
WaveletOLS-db-7	700.4352
Wavelet robust M-estimation –bior-1.1	**581.4640**
Wavelet robust M-estimation –db-7	625.0698

According to the results in Table (4), the proposed hybrid method (Wavelet robust M-estimation-bior-1.1) has a lower (RMSE) than the method (Wavelet OLS).

4 .Counclusion

1- Wavelet shrinkage filters could be used to solve the problem of noise or outliers while calculating multiple linear regression model.

2- The hybrid proposed method (wavelet robust M-estimation) is better than classical method (wavelet OLS) and more accurate.

3- When it came to estimating multiple linear regression, the wavelet filter (Haar) was the best in terms of types and levels of wavelet.

4- The RMSE of model estimated using the wavelet robust M-estimation-bio1.1 is less than in all cases (wavelet robust M-estimation-db7).

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ڕێگای تێکەڵبووی پێشنیارکراو بۆ کەمبوونی شەپۆل بەبەکارهێنانی مۆدێلی فرە هێڵی لێژی بەهێز (لێکۆڵینەوەیەکی لاسایی)

پـوخـتـە:

لەم توێژینەوەیەدا هەستاین بە خستنە ڕووی شێوازی تێکەل بووی پێشنیار کراو بۆ کەمبونی شەپۆل بەبەکارهێنانی مۆدیلی فرە هێڵی لێژی بە هێز پاشان بەراوورد کردنی لەگەڵ شێوازی کلاسیکی وە بۆ بەراووردکردنیان پشتمان بەست بە پێوەری (RMSE) . وە شێوازەی لێکۆڵینەوەیەکی لاساییمان جێ بەجێ کرد لەسەر دابەش بووی کوشی بەرێژەیەك داتای دروست کراومان پیس. دەرئەنجام گەیشتینە ئەوەی کە شێواری تێکەل بووی پێشنیار کراو بۆ کەمبونی شەپۆل بەبەکارهێنانی مۆدیلی فرە هێڵی لێژی بە هێز ئەنجامەی باشتربوو لە شێوازی کلاسیکی بۆ داتای لێکۆڵینەوەیەکی لاسایی و داتای راستەقینە .

طريقة هجينة مقترحة للتقليص المويجي مع M-الحصينة لانموذج الانحدار الخطي المتعدد(دراسة محاكاة)

الملخص:

تم في هذا الدراسة اقتراح طريقة هجينة للتقليص المويجي مع M- الحصينة ومقارنها مع الطريقة التقليدية المويجي المربع الصغرى الاعتيادية (Wavelet OLS) في تقليل مشكلة التلوث او قيم الشاذة في تقدير معلمات انموذج الانحدار الخطي المتعدد اعتماداً على جذر متوسط الخطاء التربيعي(RMSE) و ذلك من خلال تجارب المحاكاة والبيانات الحقيقية التي ، وتوصلت نتائج الدراسة الى افضلية الطريقة المقترحة مقارنةً مع الطريقة التقليدية و لذلك ، يوصى باستخدام الطريقة المقترحة لتقليل مشكلة قيم الشاذة وإزالة الضوضاء.